Position Reconstruction (PRex)

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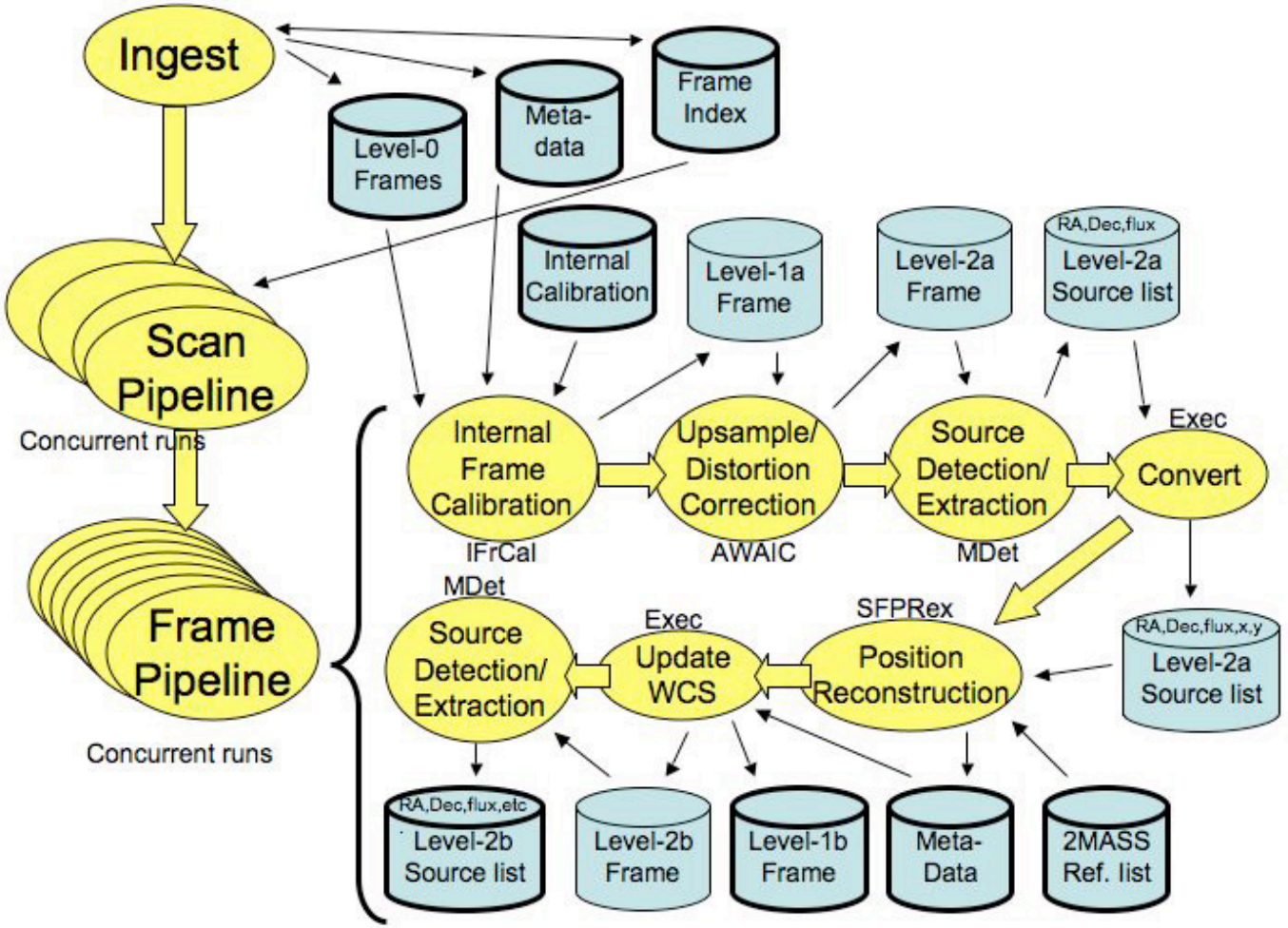
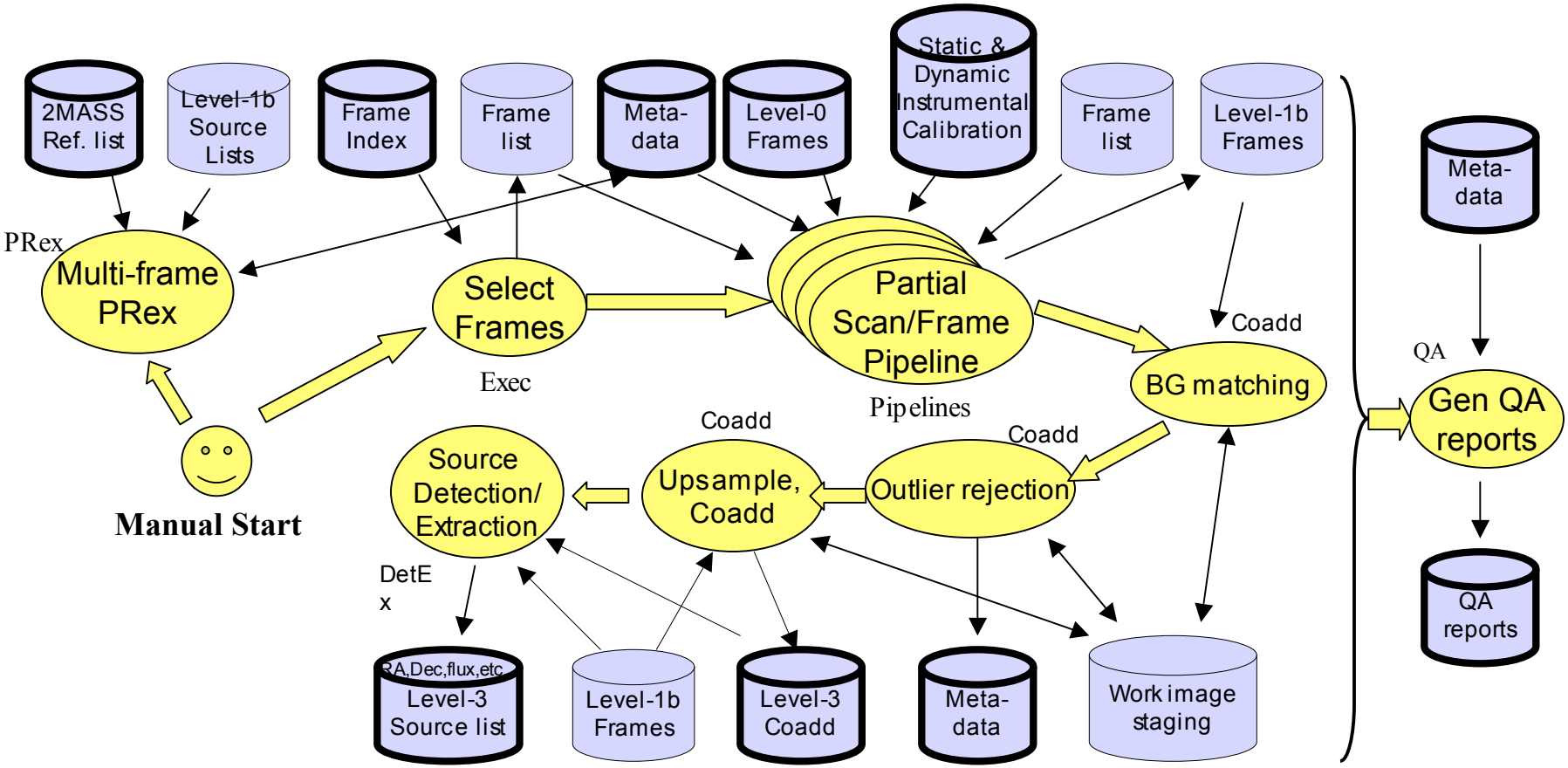
Position of SFPRex in Frame Pipeline



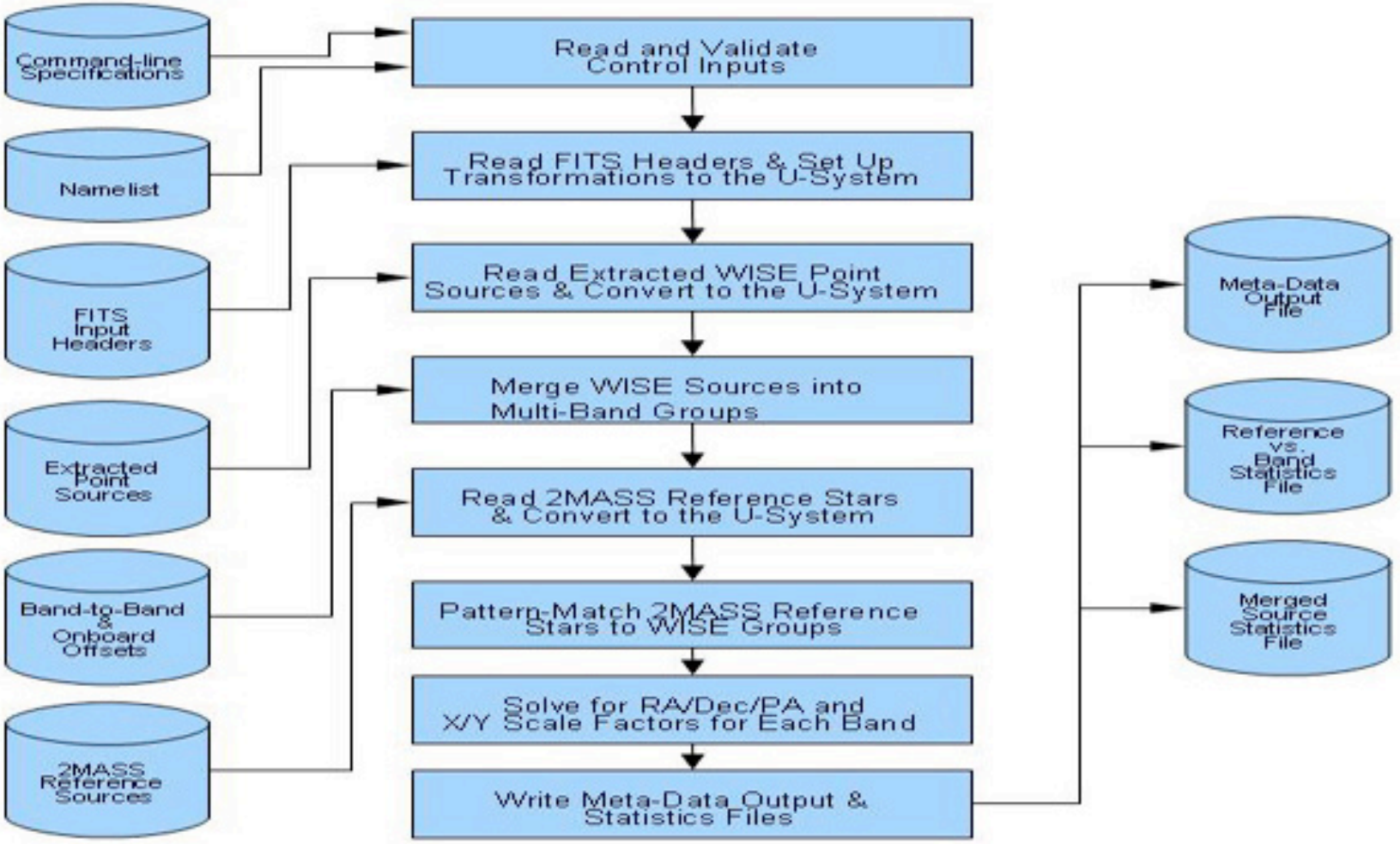
Position of MFPRex in Pipeline



PRex Purpose & Related Requirements

- **Purpose**: Reconstruct Frame Position (RA & Dec), Orientation (PA) and Scale Factors (sfx &sfy) for Each of the Four Band-Frames Making Up a Single Frameset (along with uncertainties) Sufficient to Meet Related Requirements
- **Related Requirements**:
  - WISE catalog RMS of 0.5 asec per axis for SNR>20 in one or more bands (L4WSDC-014)
  - WISE catalog to contain uncertainties for each source (L4WSDC-018)
  - Will use 2MASS All-Sky PSC as astrometric reference catalog (L4WSDC-046)
  - Will provide QA sufficient to validate (L4WSDC-062)
  - WISE catalog will provide equatorial coordinates (J2000 - ICRS) (L4WSDC-017 & L4WSDC-078)

Overall SFPRex Functional Flow Diagram



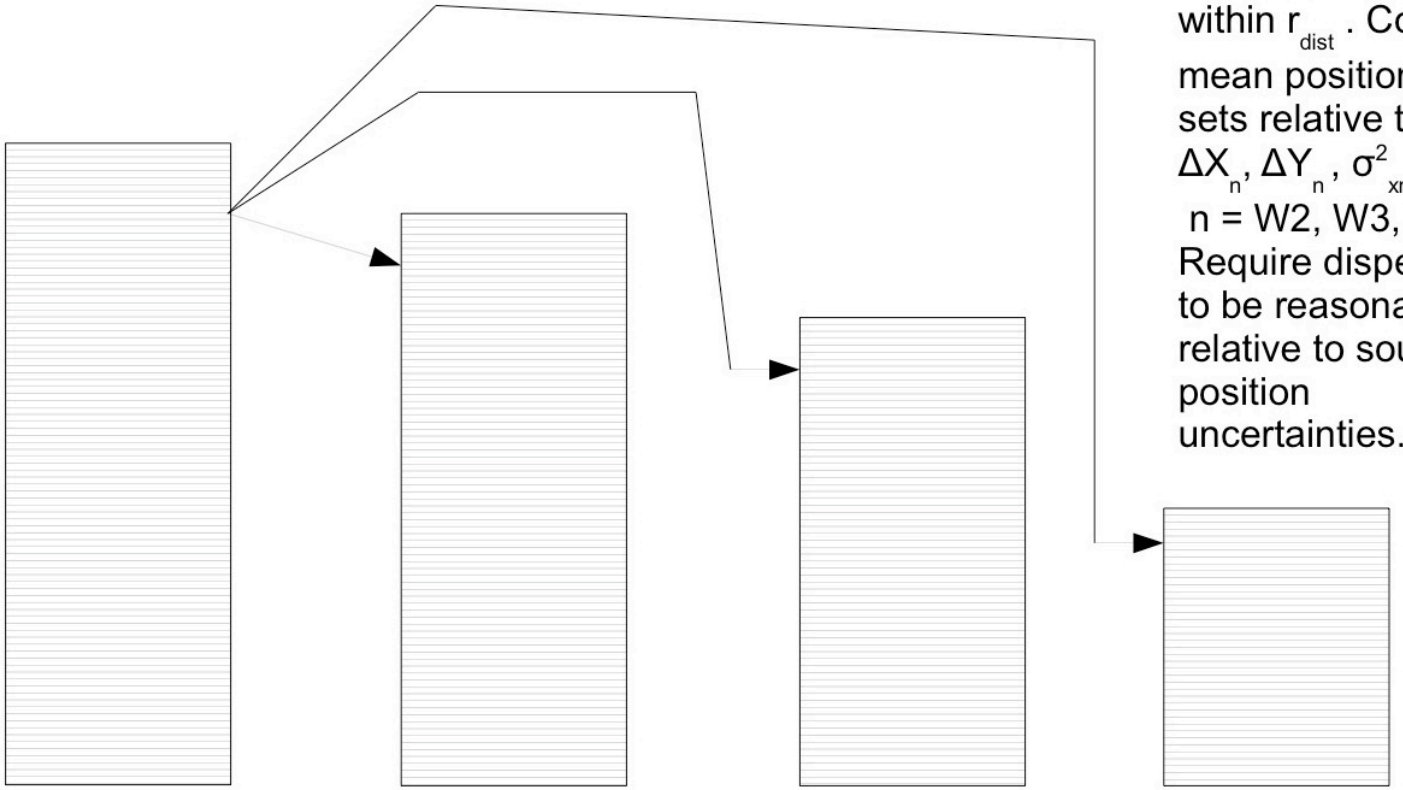
Converting Band Frame Positions to U-System

$$\begin{pmatrix} {}^{i}X_u \\ {}^{i}Y_u \end{pmatrix} = \begin{pmatrix} {}^{i}X_{u0} \\ {}^{i}Y_{u0} \end{pmatrix} + \begin{pmatrix} \cos({}^{i}\theta) & \sin({}^{i}\theta) \\ -\sin({}^{i}\theta) & \cos({}^{i}\theta) \end{pmatrix} \begin{pmatrix} {}^{i}s_x & {}^{i}X_f \\ {}^{i}s_y & {}^{i}Y_f \end{pmatrix}$$

X_{uo}	X offset in arcsec of band-frame origin from U-system origin
Y_{u0}	Y offset in arcsec of band-frame origin from U-system origin
θ	Rotation in degrees of band-frame axes relative to U-system
s_x	X scale factor to convert from band-frame to U-system
s_y	Y scale factor to convert from band-frame to U-system

Frame Adjustment - 1

For each W1 detection, find W2, W3, & W4 detections within r_{dist}. Reject all detections having more than 1 match within r_{dist}. Compute mean position off-sets relative to W1, ΔX_n, ΔY_n, σ^2_{xn}, σ^2_{yn}, n = W2, W3, W4. Require dispersions to be reasonable relative to source position uncertainties.



Frame Adjustment - 2

- Must have at least MinMatch (3) matches per band (if not, expand r_{dist} by 10% and try again, up to 100 times)
- RMS Dispersions must be <= Z*RMS Source Uncertainties (if not, repeat with cases rejected when offsets > Z*RMS Dispersion from 1st pass)
- Mean position offsets are subtracted from source coordinates when computing χ^2 for merge-group match test only
- Mean position offsets are returned for use in band-frame position correction

Merge Group Generation - 1 (Internal PRex BandMerger)

All cross-band pairings of sources are tested for position matching via 2-D χ^2 test

For source #m in band i and source #n in band j:

$$\Delta X = ({}^{i}X_m - \delta x_i) - ({}^{j}X_n - \delta x_j)$$

$$\Delta Y = ({}^{i}Y_m - \delta y_i) - ({}^{j}Y_n - \delta y_j)$$

$$\vec{\Delta} \equiv (\Delta X, \Delta Y)$$

$$\Omega = {}^{i}\Omega_m + {}^{j}\Omega_n = \begin{pmatrix} {}^{i}V_{xm} & {}^{i}V_{xym} \\ {}^{i}V_{xym} & {}^{i}V_{ym} \end{pmatrix} + \begin{pmatrix} {}^{j}V_{xn} & {}^{j}V_{xyn} \\ {}^{j}V_{xyn} & {}^{j}V_{yn} \end{pmatrix} \equiv \begin{pmatrix} V_x & V_{xy} \\ V_{xy} & V_y \end{pmatrix}$$

$$W \equiv \Omega^{-1} = \frac{1}{D}\begin{pmatrix} V_y & -V_{xy} \\ -V_{xy} & V_x \end{pmatrix} \equiv \begin{pmatrix} W_x & W_{xy} \\ W_{xy} & W_y \end{pmatrix}, \quad D \equiv V_x V_y - V_{xy}^2$$

$$\chi^2 = \vec{\Delta} W \vec{\Delta}^T = W_x \Delta X^2 + W_y \Delta Y^2 + 2W_{xy} \Delta X \Delta Y$$

$$= \frac{V_y \Delta X^2 + V_x \Delta Y^2 - 2V_{xy} \Delta X \Delta Y}{D}$$

Source pair is a match if $\chi^2 < \chi^2_{max}$ (6; implies 5% real matches sacrificed for reliability)

Merge Group Generation - 2

- Each source in a merge group must match at least one other source in that group
- No source may have more than one match in another band; if any does, all sources are rejected from membership in any group
- Merge groups are bookkept in arrays of source indexes, MG(N_g,4); for example, if group #17 consists of W1 source #21 and W3 source #9, then MG(17,n), n = 1 to 4, is {21, 0, 9, 0}
- Denoting the members of a given group as {G}, position refinement is performed to obtain the group position and uncertainties as follows:

$$W = \sum_{i \in \{G\}} {}^{i}\Omega^{-1} = \sum_{i \in \{G\}} \frac{1}{{}^{i}V_x\,{}^{i}V_y - {}^{i}V_{xy}^{2}} \begin{pmatrix} {}^{i}V_y & -{}^{i}V_{xy} \\ -{}^{i}V_{xy} & {}^{i}V_x \end{pmatrix}$$

$$\Omega_{refined} = W^{-1}$$

$$\begin{pmatrix} X \\ Y \end{pmatrix}_{refined} = \Omega_{refined} \sum_{i \in \{G\}} {}^{i}\Omega^{-1} \begin{pmatrix} {}^{i}X \\ {}^{i}Y \end{pmatrix}$$

Selection of 2MASS Stars for Use as Astrometric References

- Current Selection Criteria
  - Taken from All-Sky Release Point Source Catalog (PSC)
  - Clean (Unconfused, not a known asteroid, ..)
  - Have Ks magnitudes between 5.5 and 12.0
  - Results in 30 million sources
  - Counts per frame vary with sky position (average 446)
- Uncertainty in % 2MASS Sources Visible in each WISE Band
- Smarter Selection Possible Without Loss in Reference Count
  - Can increase Ks magnitude range while retaining 0.1 arc-second accuracy
  - Predict how 2MASS stars might map to WISE bands
  - Avoid use of 2MASS stars with large proper motions (as determined from extant proper motion catalogs)

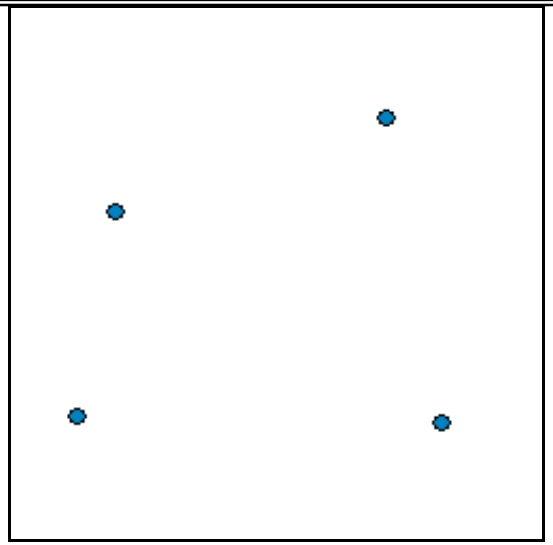
Pattern Matcher

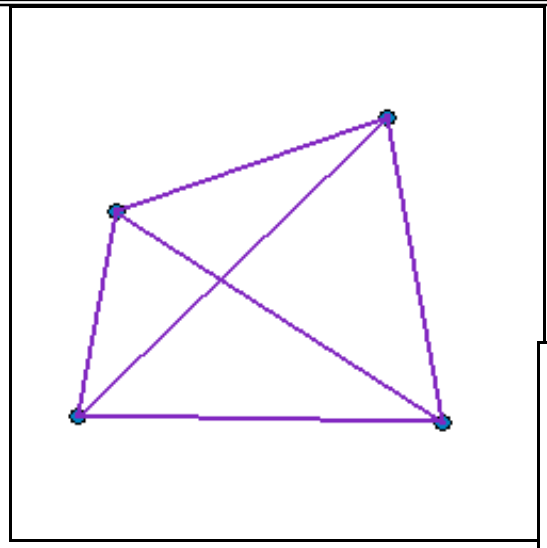
- Set up Separation Bars Between Pairs of Sources
  - Consider all possible bars within 2MASS & WISE Merge Group sets
    - Brightest *ldepth* (set to 99) sources
    - Minimun separation of *sepmin* (set to 60 asec)
- Set up Bar Match Candidates Between 2MASS and WISE
  - Max difference in PA between candidate bar pairs of *toldpa* (set to 4000 asec)
  - Max difference from 1.0 of bar length ratio = *tolds* (set to .015)
- Force Exact Alignment of Candidate Separation Bar Pairs
  - Use Two-Peg approach from 2MASS to Torque WISE Merge Group Set
  - Compute and save source match counts & required adjustments ΔX, ΔY, θ, sf
- Evaluate Probability That All Source Matches Spurious
- Option controlled by *useals:*
  - Compute trimmed average of adjustments for bar pairs with best source match counts
- Option controlled by *twkmch*:
  - Do 5-parameter fit using all source match => dx, dy, $d\theta$, ds_x, ds_y

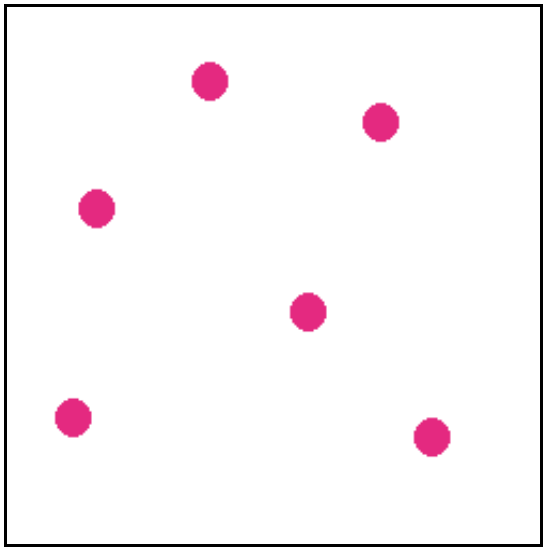
Separation Bar Matching (Simplified Illustration)

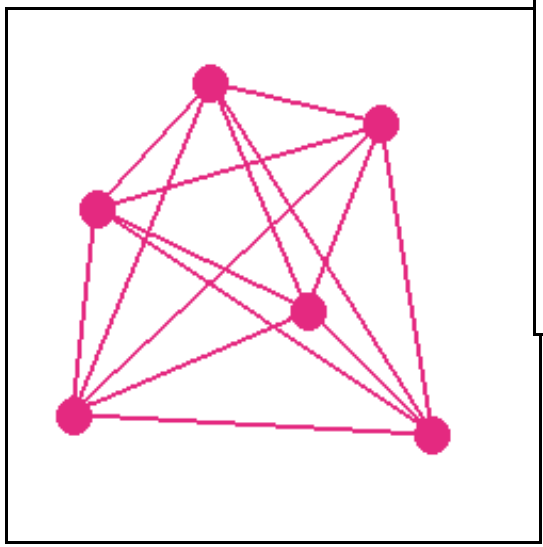
2MASS

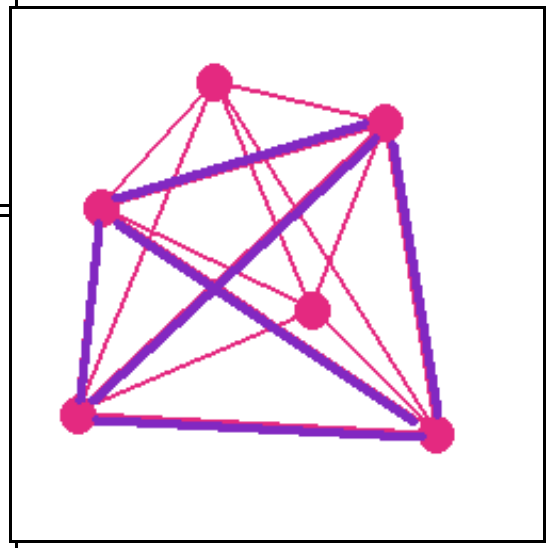
WISE





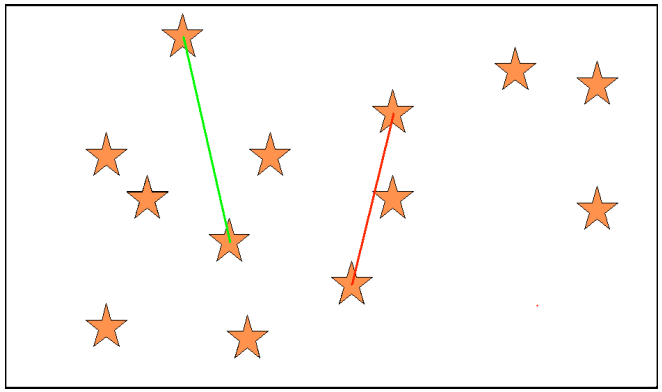




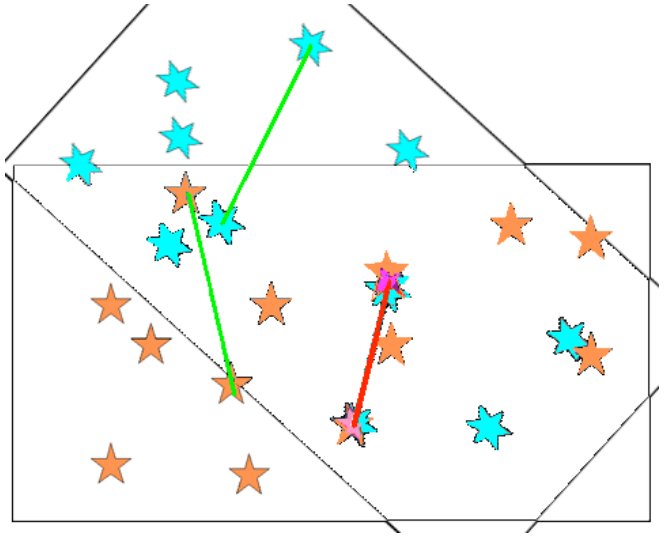


WISE & 2MASS

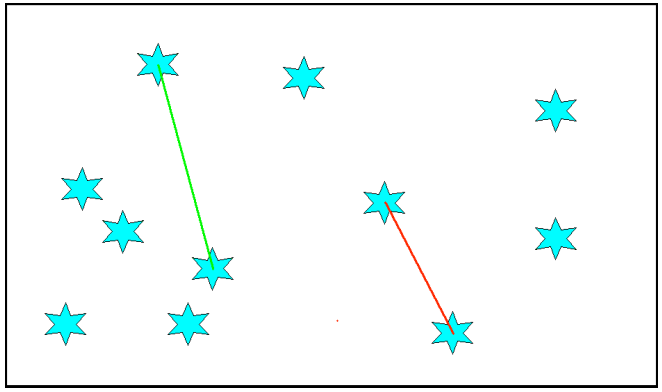
Two Peg Source Match Counts Identify Good Sep-Bar Match



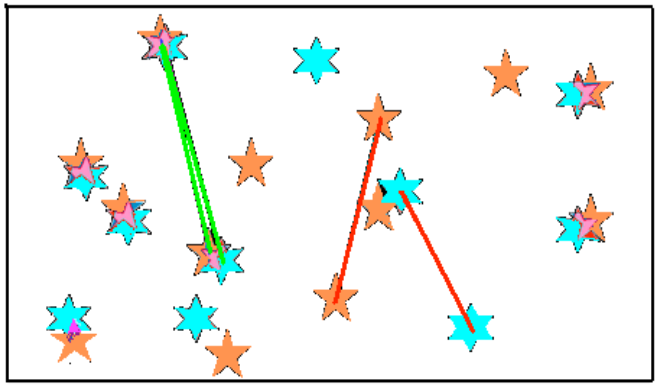
WISE Merged Groups



Source Match Count => Bad Bar Match



2MASS Reference Stars



Source Match Count => Good Bar Match

Small Adjustment Parameters

- Five small adjustment parameters per band-frame

  Translation (2) : $\Delta X, \Delta Y$

  Rotation (1): $\Delta\theta$

  Scale (2): ds_x, ds_y

Four Bands => Total of 20 small adjustment parameters

$$P_1 \equiv {}^1\Delta X, P_2 \equiv {}^1\Delta Y, P_3 \equiv {}^1\Delta\theta, P_4 \equiv {}^1ds_x, P_5 \equiv {}^1ds_y$$

$$P_6 \equiv {}^2\Delta X, P_7 \equiv {}^2\Delta Y, P_8 \equiv {}^2\Delta\theta, P_9 \equiv {}^2ds_x, P_{10} \equiv {}^2ds_y$$

$$P_{11} \equiv {}^3\Delta X, P_{12} \equiv {}^3\Delta Y, P_{13} \equiv {}^3\Delta\theta, P_{14} \equiv {}^3ds_x, P_{15} \equiv {}^3ds_y$$

$$P_{16} \equiv {}^4\Delta X, P_{17} \equiv {}^4\Delta Y, P_{18} \equiv {}^4\Delta\theta, P_{19} \equiv {}^4ds_x, P_{20} \equiv {}^4ds_y$$

Setting Up χ^2 Minimization Equations for FSFitR

$$\chi^2 = \chi^2_{ww} + \chi^2_{wr} + \chi^2_{aw} + \chi^2_{ar}$$

$\chi^2_{ww} = \chi^2$ sum of all WISE - to - WISE (band - to - band) differences

$\chi^2_{wr} = \chi^2$ sum of all WISE - to - Ref differences

$\chi^2_{aw} = \chi^2$ sum of parameter changes from *apriori* values

$\chi^2_{ar} = \chi^2$ sum reflecting changes in *apriori* band - to - band alignments

$$\frac{\partial \chi^2}{\partial P_n} = 0, \; n = 1 \; to \; 20$$

Provides 20 Equations in 20 Unknowns

Assigning Uncertainties to the 20 Parameters

The matrix equation for the χ^2 minimization takes the form :

$$Ax = b$$

The equation can be solved by taking the inverse of matrix A

$$x = A^{-1}b$$

Fortunately for a linear system the the error covariance matrix Ω_p is just A inverse

$$\Omega_p = A^{-1}$$

Taking square - roots of the diagonal elements of Ω_p provides sigmas for all parameters

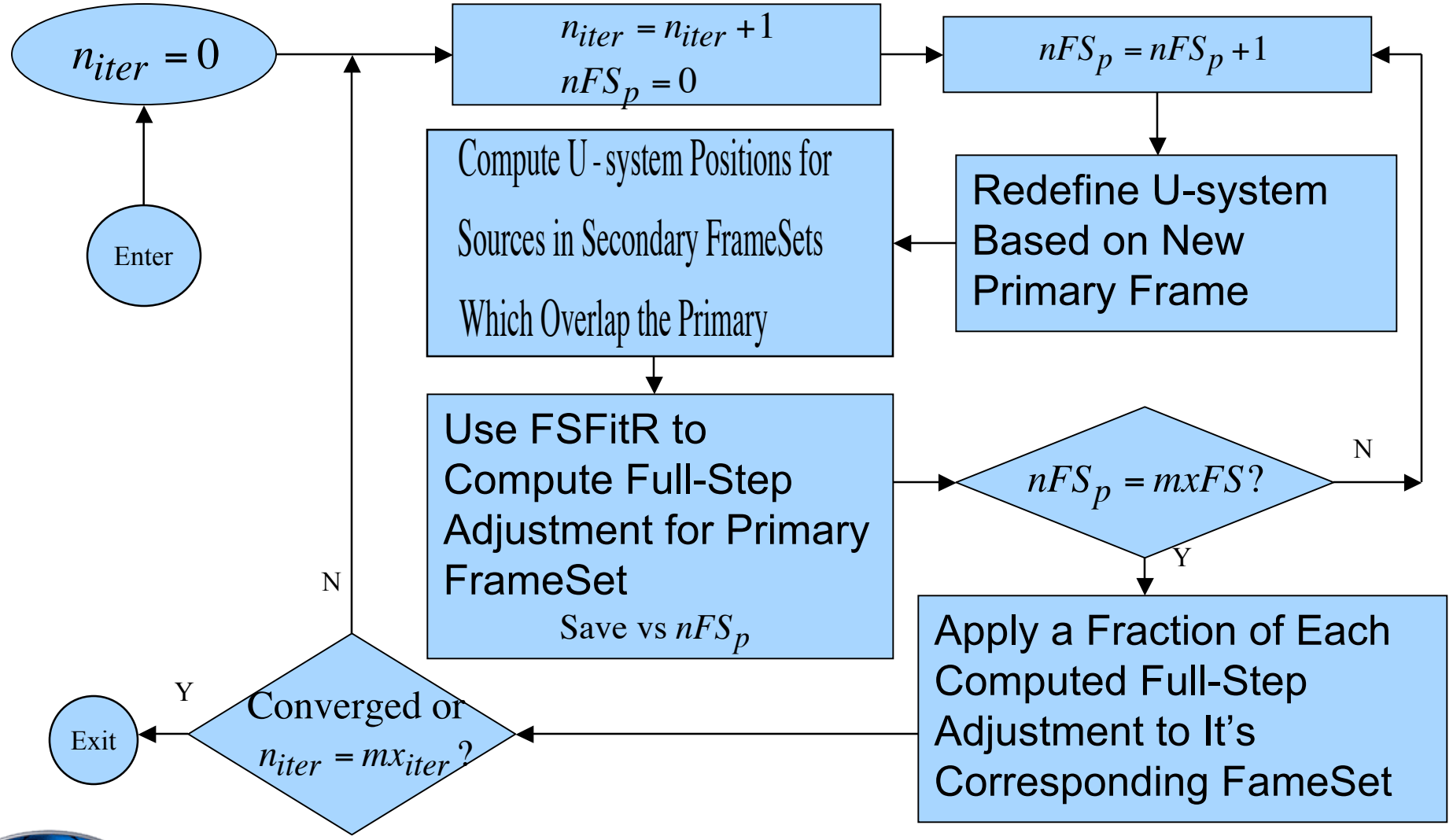
Testing and Parameter Tuning

- Testing:
  - Robustness testing of pattern matcher
    - High and low source densities
    - Large *apriori* position errors
    - Sensitivity to brightest WISE sources matching brightest 2MASS
  - Testing of 20-parameter frameset fitter
    - Explore band-to-band and band-to-ref count parameter space
    - Consider effect of no band-to-ref counts for band 4
- Parameter Tuning:
  - Set pattern match parameters such that
    - Unlikely a good match will be rejected
    - Very unlikely a bad match will be accepted
  - Set K_{aw} weighting factors

SFPRex Peer Review Comments

- Careful selection of 2MASS subset to use as reference
  - -Select stars most likely to show in WISE Bands
  - -Avoid stars with high proper motions
    (as identified per extant proper motion catalogs)
  - -Consider using brightest stars for pattern matching only
  - -Drop stars from 2nd processing which are rejected on 1st
- High priority to accurate determination of distortion model
- Consider how to handle long term scale changes
- Consider recourse with loss of bands 1 and 2
- Test fields to check for latent systematics in reconstruction

Functional MFPRex FlowChart



CPU Time Sensitive MFPRex Design Issues

- Use of FSFitR for primary frameset fitter
  - -Provides full flexibility (any subset of the 20 parameters)
  - -Could recover low source count W3 & W4 frames
  - -Already developed & tested in SFPRex

- Redefinition of U-system for each frameset
  - -Allows unlimited footprint size (up to 4π steradians)
  - -Cartesian system never covers more than ~1 degree

PRex Development Schedule

- May 01, 2008: All liens removed from SFPRex (except proper motions)
  MFPRex design complete
  (issues dependent on CPU timing tests remain)
- June 01, 2008: 2MASS proper motion modeling available
- July 17, 2008 (WSDS v1): “Complete” SFPRex code delivered
  “Prototype” MFPRex code delivered
- Sept 15, 2008: Decision on CPU time sensitive MFPRex design issues
- Dec 01, 2008: “Preliminary” MFPRex code delivered
- Feb 28, 2009 (WSDS v2): "Mature" SFPRex code delivered
  “Complete” MFPrex code delivered
- Aug 04, 2009 (WSDS v3): “Mature” MFPrex code delivered

Liens/Concerns

- The 20-parameter fit routine (FSFitR) not yet installed
- Outlier detection and rejection algorithms have not yet been designed.
- SIS's have not yet been written for output files.
- Refinement of pattern match efficency & acceptance criteria
- How to handle lack of proper motions in 2MASS undecided
- Uncertainties coming out of pattern matcher (*apriori* inputs to FSFitR)
- Prediction of which 2MASS stars likely to show up in W1, W2, W3 & W4
- Predictions of % W1 sources also found in W2, W3 & W4;
  % W2 sources also found in W3 &W4; % W3 sources also in W4
- Recourse if bands one and two are both missing
- Verification via testing of CPU time sensitive MFPRex design issues
  -Use of FSFitR for primary frameset fitter
  -Redefinition of U-system for each frameset
- Fallback position if MFPRex CPU time test results unfavorable