



National Aeronautics and Space
Administration
Jet Propulsion Laboratory
California Institute of Technology



PRex

Position Reconstruction (PRex)

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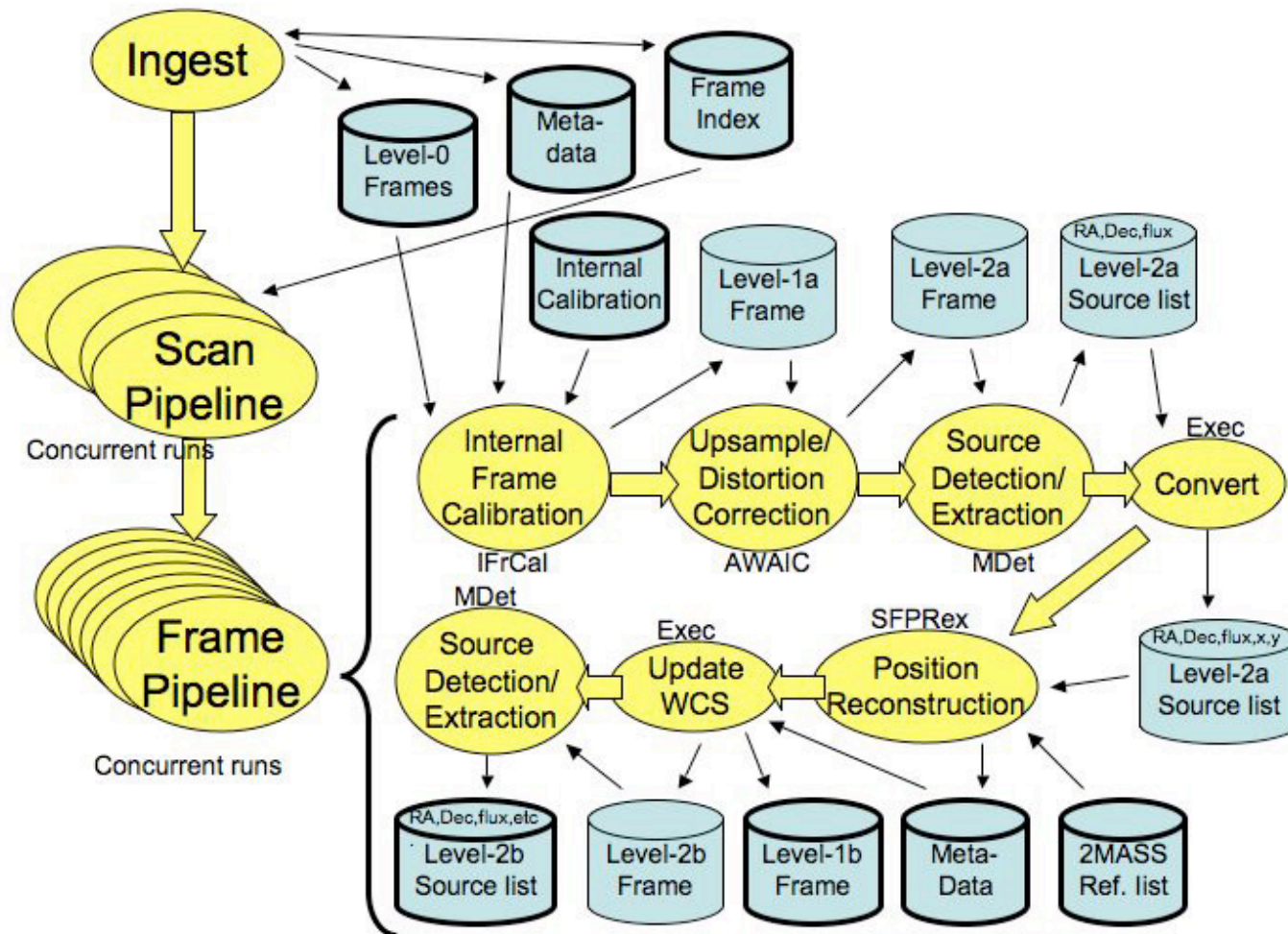
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HLM/JWF



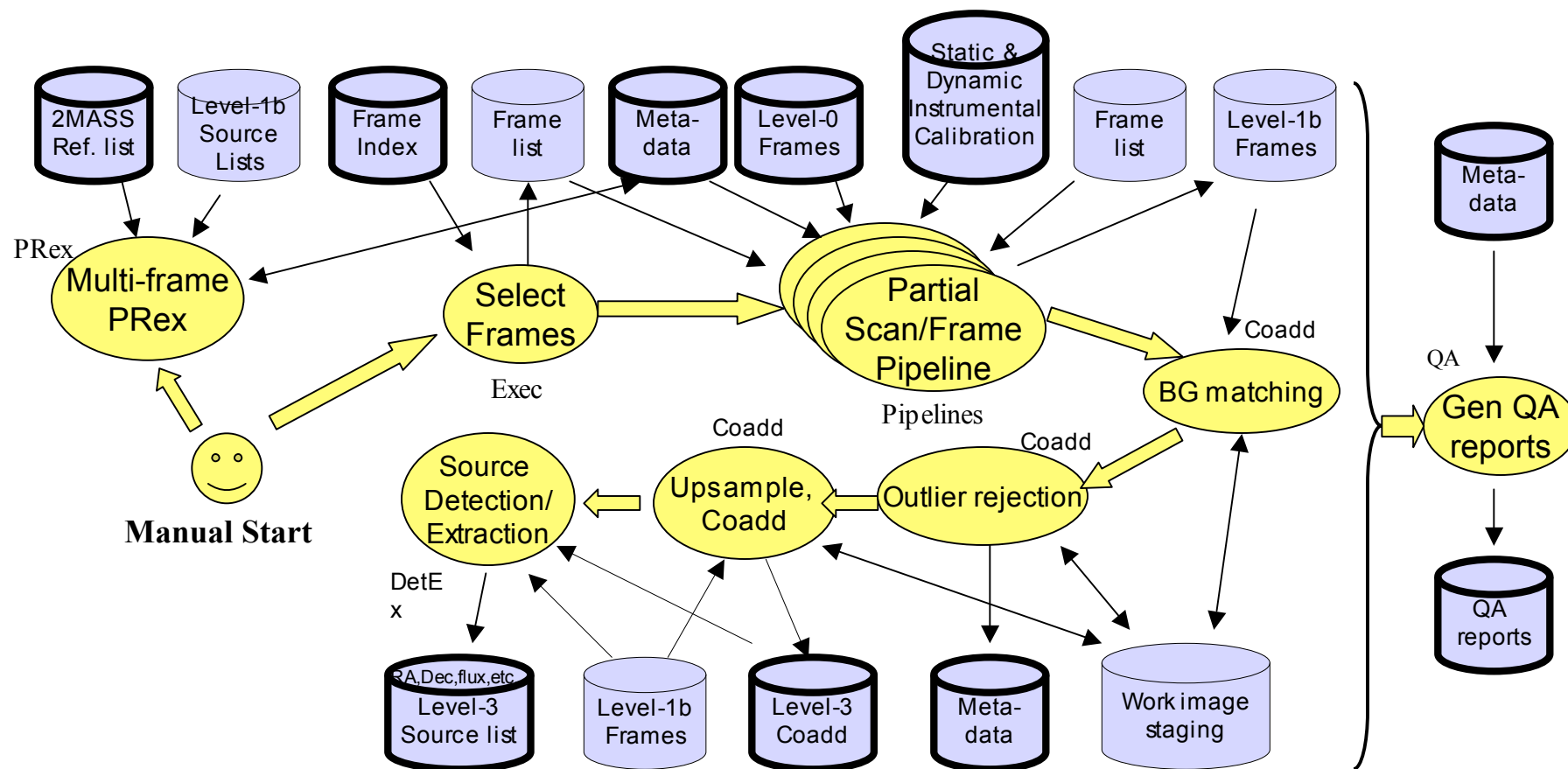
Position of SFPRex in Frame Pipeline



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Position of MFPRex in Pipeline



PRex Purpose & Related Requirements



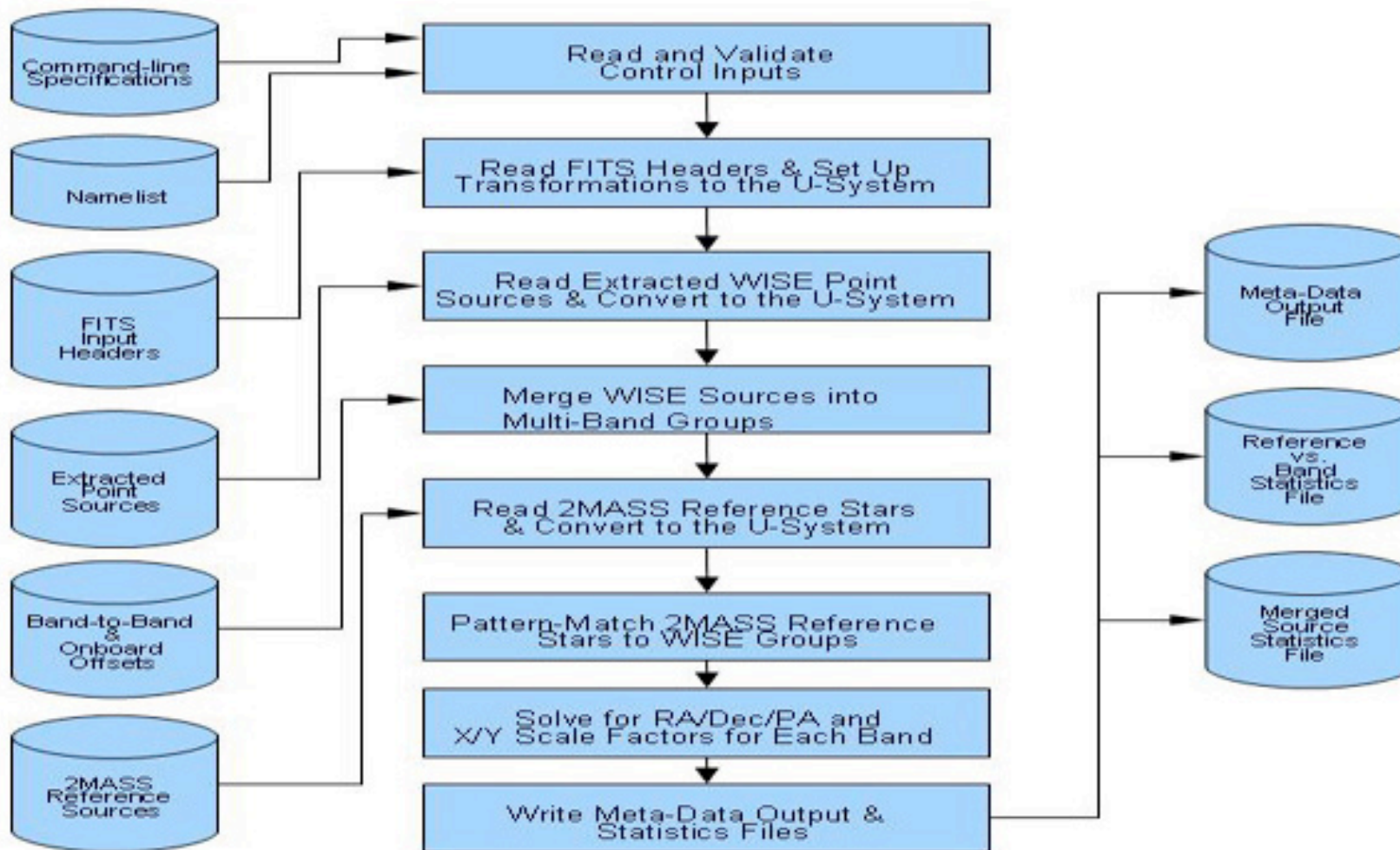
- **Purpose:** Reconstruct Frame Position (RA & Dec), Orientation (PA) and Scale Factors (sfx &sfy) for Each of the Four Band-Frames Making Up a Single Frameset (along with uncertainties) Sufficient to Meet Related Requirements
- **Related Requirements:**
 - WISE catalog RMS of 0.5 asec per axis for SNR>20 in one or more bands (L4WSDC-014)
 - WISE catalog to contain uncertainties for each source (L4WSDC-018)
 - Will use 2MASS All-Sky PSC as astrometric reference catalog (L4WSDC-046)
 - Will provide QA sufficient to validate (L4WSDC-062)
 - WISE catalog will provide equatorial coordinates (J2000 - ICRS) (L4WSDC-017 & L4WSDC-078)



Overall SFPReX Functional Flow Diagram



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Converting Band Frame Positions to U-System

$$\begin{pmatrix} {}^iX_u \\ {}^iY_u \end{pmatrix} = \begin{pmatrix} {}^iX_{u0} \\ {}^iY_{u0} \end{pmatrix} + \begin{pmatrix} \cos({}^i\theta) & \sin({}^i\theta) \\ -\sin({}^i\theta) & \cos({}^i\theta) \end{pmatrix} \begin{pmatrix} {}^iS_x & {}^iX_f \\ {}^iS_y & {}^iY_f \end{pmatrix}$$

X_{u0}	X offset in arcsec of band-frame origin from U-system origin
Y_{u0}	Y offset in arcsec of band-frame origin from U-system origin
θ	Rotation in degrees of band-frame axes relative to U-system
S_x	X scale factor to convert from band-frame to U-system
S_y	Y scale factor to convert from band-frame to U-system

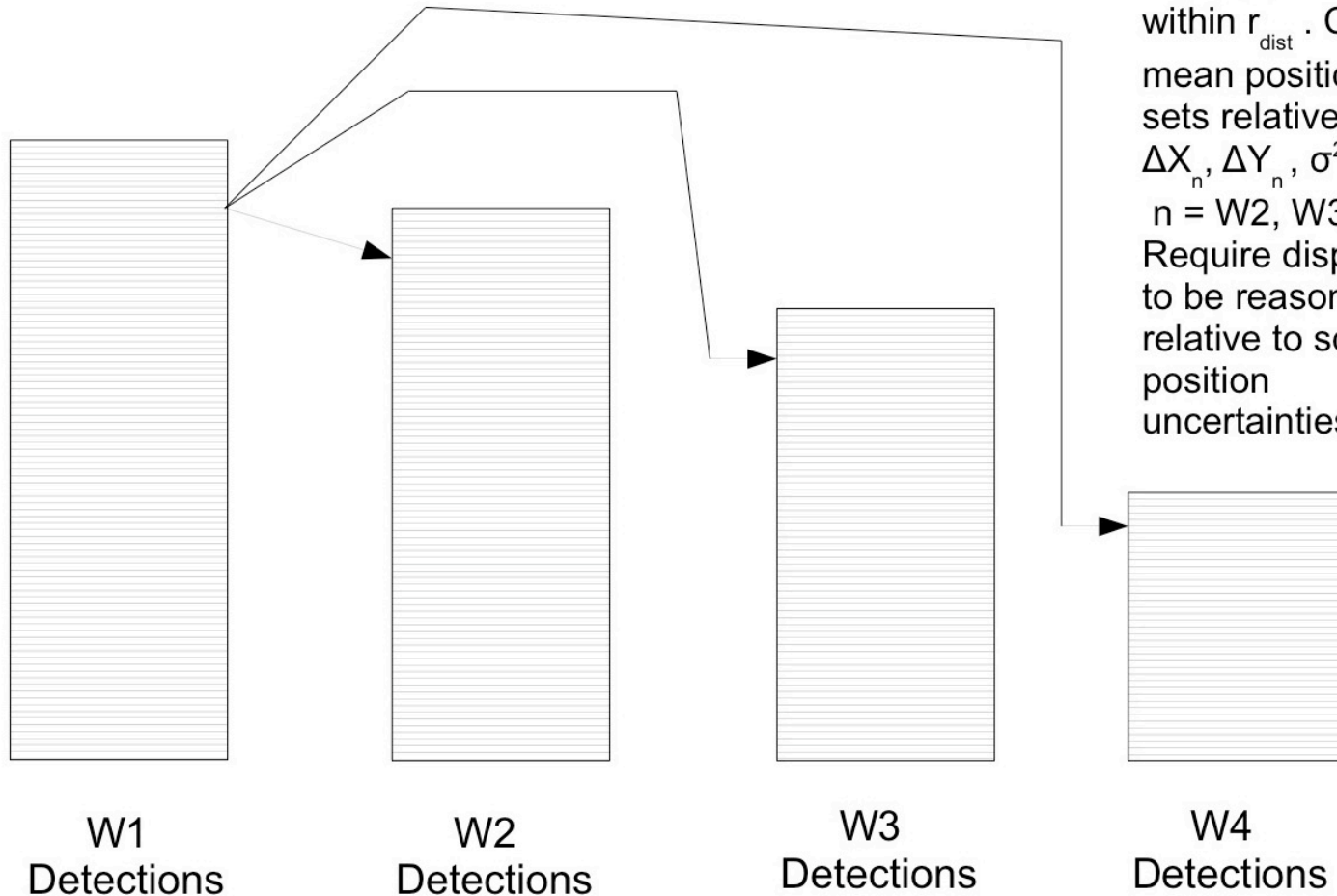


Frame Adjustment - 1



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For each W1 detection, find W2, W3, & W4 detections within r_{dist} . Reject all detections having more than 1 match within r_{dist} . Compute mean position offsets relative to W1, $\Delta X_n, \Delta Y_n, \sigma_{x_n}^2, \sigma_{y_n}^2$, $n = \text{W2, W3, W4}$. Require dispersions to be reasonable relative to source position uncertainties.





Frame Adjustment - 2



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- Intended primarily for IOC
- Must have at least MinMatch (3) matches per band (if not, expand r_{dist} by 10% and try again, up to 100 times)
- RMS Dispersions must be $\leq Z \cdot \text{RMS Source Uncertainties}$ (if not, repeat with cases rejected when offsets $> Z \cdot \text{RMS Dispersion}$ from 1st pass)
- Mean position offsets are subtracted from source coordinates when computing χ^2 for merge-group match test only
- Mean position offsets are returned for use in band-frame position correction





Merge Group Generation - 1 (Internal PRex BandMerger)



All cross-band pairings of sources are tested for position matching via 2-D χ^2 test

For source #m in band i and source #n in band j:

$$\Delta X = ({}^iX_m - \delta x_i) - ({}^jX_n - \delta x_j)$$

$$\Delta Y = ({}^iY_m - \delta y_i) - ({}^jY_n - \delta y_j)$$

$$\vec{\Delta} \equiv (\Delta X, \Delta Y)$$

$$\Omega = {}^i\Omega_m + {}^j\Omega_n = \begin{pmatrix} {}^iV_{xm} & {}^iV_{xym} \\ {}^iV_{xym} & {}^iV_{ym} \end{pmatrix} + \begin{pmatrix} {}^jV_{xn} & {}^jV_{xyn} \\ {}^jV_{xyn} & {}^jV_{yn} \end{pmatrix} \equiv \begin{pmatrix} V_x & V_{xy} \\ V_{xy} & V_y \end{pmatrix}$$

$$W \equiv \Omega^{-1} = \frac{1}{D} \begin{pmatrix} V_y & -V_{xy} \\ -V_{xy} & V_x \end{pmatrix} \equiv \begin{pmatrix} W_x & W_{xy} \\ W_{xy} & W_y \end{pmatrix}, \quad D \equiv V_x V_y - V_{xy}^2$$

$$\begin{aligned} \chi^2 &= \vec{\Delta}^T W \vec{\Delta} = W_x \Delta X^2 + W_y \Delta Y^2 + 2W_{xy} \Delta X \Delta Y \\ &= \frac{V_y \Delta X^2 + V_x \Delta Y^2 - 2V_{xy} \Delta X \Delta Y}{D} \end{aligned}$$

Source pair is a match if $\chi^2 < \chi^2_{\max}$ (6; implies 5% real matches sacrificed for reliability)





Merge Group Generation - 2



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- Each source in a merge group must match at least one other source in that group
- No source may have more than one match in another band; if any does, all sources are rejected from membership in any group
- Merge groups are bookkept in arrays of source indexes, $MG(N_g, 4)$; for example, if group #17 consists of W1 source #21 and W3 source #9, then $MG(17, n)$, $n = 1$ to 4 , is $\{21, 0, 9, 0\}$
- Denoting the members of a given group as $\{G\}$, position refinement is performed to obtain the group position and uncertainties as follows:

$$W = \sum_{i \in \{G\}} {}^i \Omega^{-1} = \sum_{i \in \{G\}} \frac{1}{{}^i V_x {}^i V_y - {}^i V_{xy}^2} \begin{pmatrix} {}^i V_y & - {}^i V_{xy} \\ - {}^i V_{xy} & {}^i V_x \end{pmatrix}$$

$$\Omega_{refined} = W^{-1}$$

$$\begin{pmatrix} X \\ Y \end{pmatrix}_{refined} = \Omega_{refined} \sum_{i \in \{G\}} {}^i \Omega^{-1} \begin{pmatrix} {}^i X \\ {}^i Y \end{pmatrix}$$





Selection of 2MASS Stars for Use as Astrometric References



- Current Selection Criteria
 - Taken from All-Sky Release Point Source Catalog (PSC)
 - Clean (Unconfused, not a known asteroid, ..)
 - Have Ks magnitudes between 5.5 and 12.0
 - Results in 30 million sources
 - Counts per frame vary with sky position (average 446)
- Uncertainty in % 2MASS Sources Visible in each WISE Band
- Smarter Selection Possible Without Loss in Reference Count
 - Can increase Ks magnitude range while retaining 0.1 arc-second accuracy
 - Predict how 2MASS stars might map to WISE bands
 - Avoid use of 2MASS stars with large proper motions
(as determined from extant proper motion catalogs)





Pattern Matcher



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- Set up Separation Bars Between Pairs of Sources
 - Consider all possible bars within 2MASS & WISE Merge Group sets
 - Brightest *ldepth* (set to 99) sources
 - Minimum separation of *sepmin* (set to 60 asec)
- Set up Bar Match Candidates Between 2MASS and WISE
 - Max difference in PA between candidate bar pairs of *toldpa* (set to 4000 asec)
 - Max difference from 1.0 of bar length ratio = *tolds* (set to .015)
- Force Exact Alignment of Candidate Separation Bar Pairs
 - Use Two-Peg approach from 2MASS to Torque WISE Merge Group Set
 - Compute and save source match counts & required adjustments $\Delta X, \Delta Y, \theta, sf$
- Evaluate Probability That All Source Matches Spurious
- Option controlled by *useals*:
 - Compute trimmed average of adjustments for bar pairs with best source match counts
- Option controlled by *twkmch*:
 - Do 5-parameter fit using all source match $\Rightarrow dx, dy, d\theta, ds_x, ds_y$

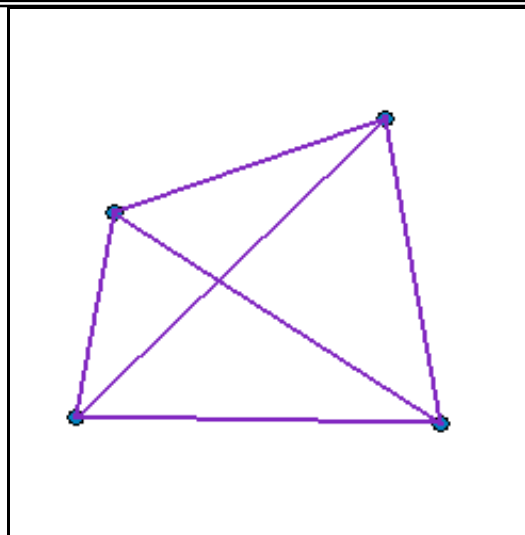
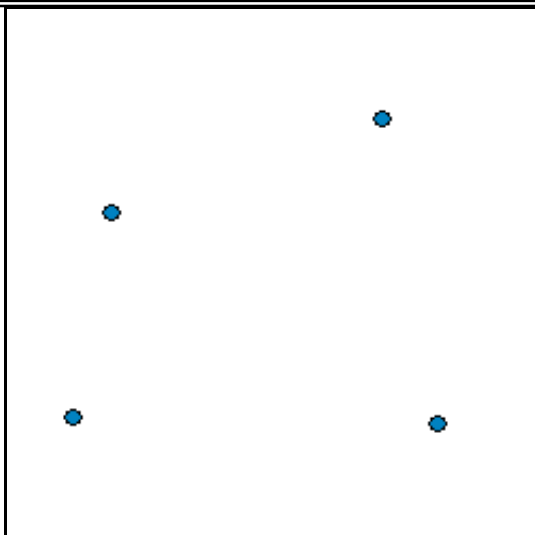


Separation Bar Matching (Simplified Illustration)

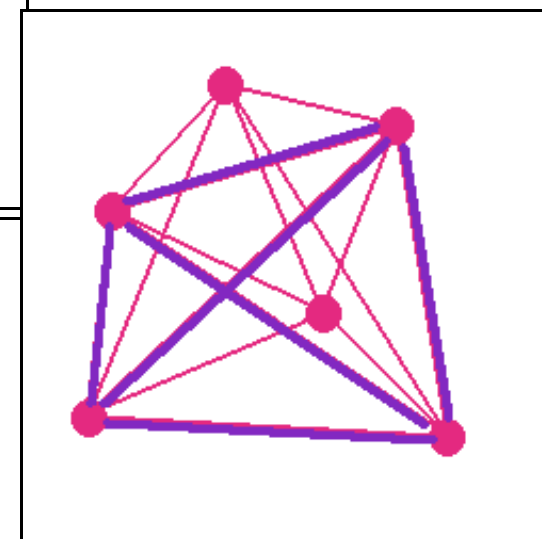
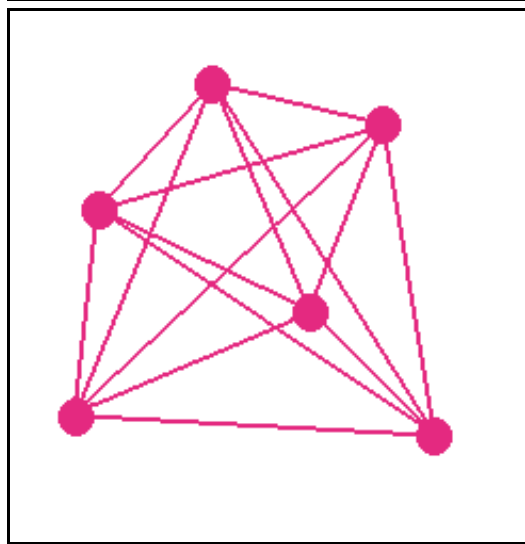
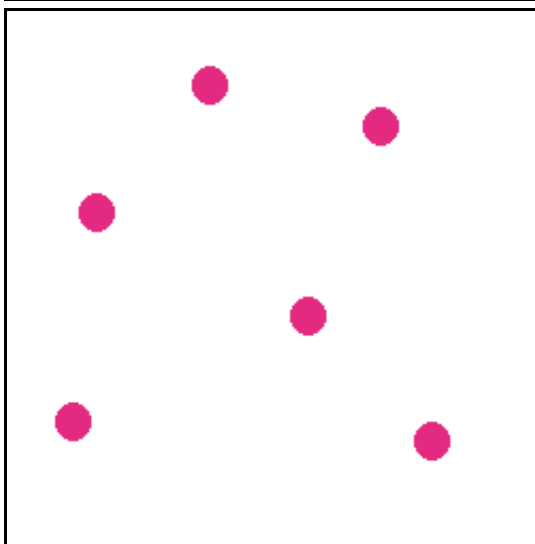


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2MASS



WISE

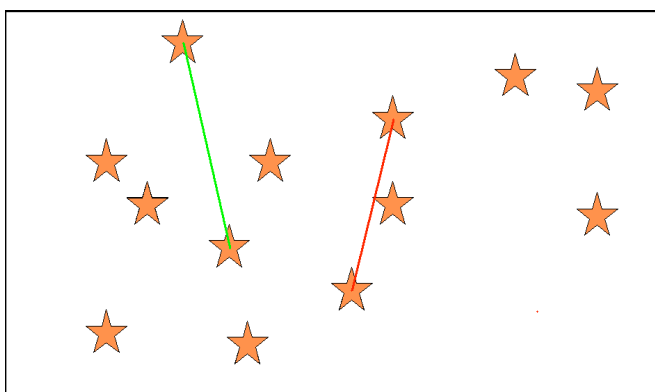


WISE & 2MASS

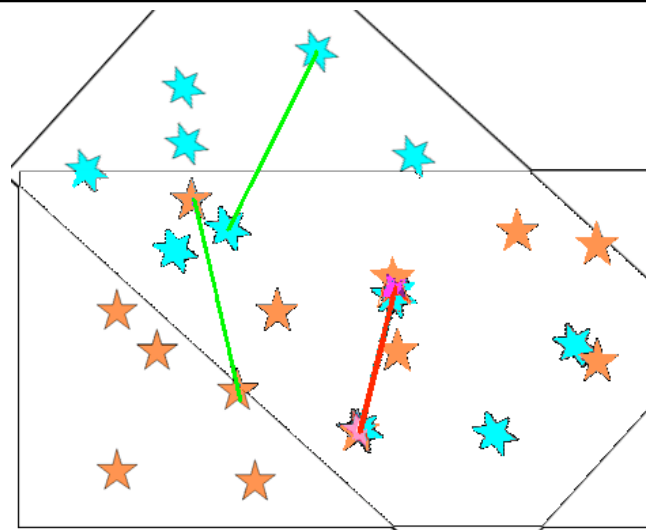
Two Peg Source Match Counts Identify Good Sep-Bar Match



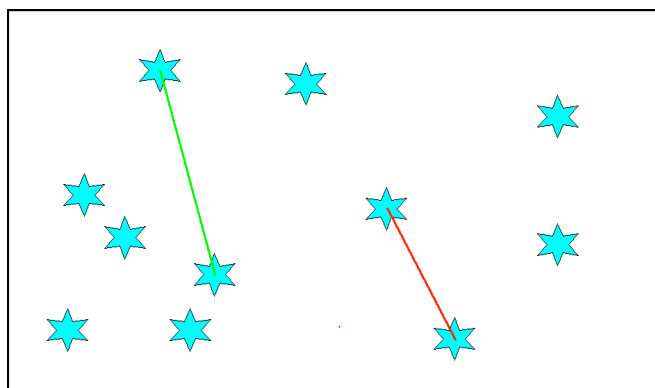
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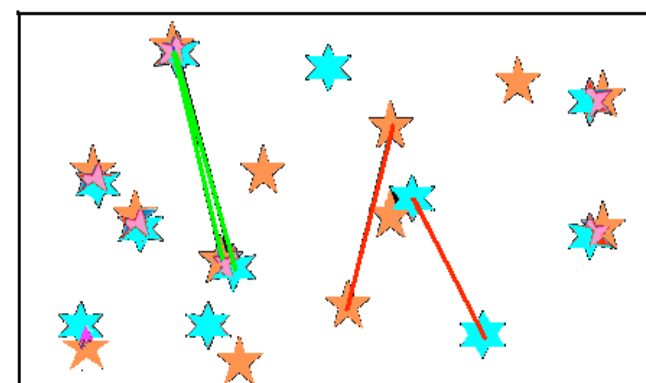
WISE Merged Groups



Source Match Count => Bad Bar Match



2MASS Reference Stars



Source Match Count => Good Bar Match



Small Adjustment Parameters



- Five small adjustment parameters per band-frame

Translation (2) : $\Delta X, \Delta Y$

Rotation (1): $\Delta \theta$

Scale (2): ds_x, ds_y

Four Bands => Total of 20 small adjustment parameters

$$P_1 \equiv {}^1\Delta X, P_2 \equiv {}^1\Delta Y, P_3 \equiv {}^1\Delta \theta, P_4 \equiv {}^1ds_x, P_5 \equiv {}^1ds_y$$

$$P_6 \equiv {}^2\Delta X, P_7 \equiv {}^2\Delta Y, P_8 \equiv {}^2\Delta \theta, P_9 \equiv {}^2ds_x, P_{10} \equiv {}^2ds_y$$

$$P_{11} \equiv {}^3\Delta X, P_{12} \equiv {}^3\Delta Y, P_{13} \equiv {}^3\Delta \theta, P_{14} \equiv {}^3ds_x, P_{15} \equiv {}^3ds_y$$

$$P_{16} \equiv {}^4\Delta X, P_{17} \equiv {}^4\Delta Y, P_{18} \equiv {}^4\Delta \theta, P_{19} \equiv {}^4ds_x, P_{20} \equiv {}^4ds_y$$



Setting Up χ^2 Minimization Equations for FSFitR

$$\chi^2 = \chi_{ww}^2 + \chi_{wr}^2 + \chi_{aw}^2 + \chi_{ar}^2$$

$\chi_{ww}^2 = \chi^2$ sum of all WISE - to - WISE (band - to - band) differences

$\chi_{wr}^2 = \chi^2$ sum of all WISE - to - Ref differences

$\chi_{aw}^2 = \chi^2$ sum of parameter changes from *apriori* values

$\chi_{ar}^2 = \chi^2$ sum reflecting changes in *apriori* band - to - band alignments

$$\frac{\partial \chi^2}{\partial P_n} = 0, n = 1 \text{ to } 20$$

Provides 20 Equations in 20 Unknowns

Assigning Uncertainties to the 20 Parameters

The matrix equation for the χ^2 minimization takes the form :

$$Ax = b$$

The equation can be solved by taking the inverse of matrix A

$$x = A^{-1}b$$

Fortunately for a linear system the the error covariance matrix Ω_p is just A inverse

$$\Omega_p = A^{-1}$$

Taking square - roots of the diagonal elements of Ω_p provides sigmas for all parameters



- Testing:
 - Robustness testing of pattern matcher
 - High and low source densities
 - Large *a priori* position errors
 - Sensitivity to brightest WISE sources matching brightest 2MASS
 - Testing of 20-parameter frameset fitter
 - Explore band-to-band and band-to-ref count parameter space
 - Consider effect of no band-to-ref counts for band 4
- Parameter Tuning:
 - Set pattern match parameters such that
 - Unlikely a good match will be rejected
 - Very unlikely a bad match will be accepted
 - Set K_{aw} weighting factors





SFPRex Peer Review Comments



- Careful selection of 2MASS subset to use as reference
 - Select stars most likely to show in WISE Bands
 - Avoid stars with high proper motions
(as identified per extant proper motion catalogs)
 - Consider using brightest stars for pattern matching only
 - Drop stars from 2nd processing which are rejected on 1st
- High priority to accurate determination of distortion model
- Consider how to handle long term scale changes
- Consider recourse with loss of bands 1 and 2
- Test fields to check for latent systematics in reconstruction

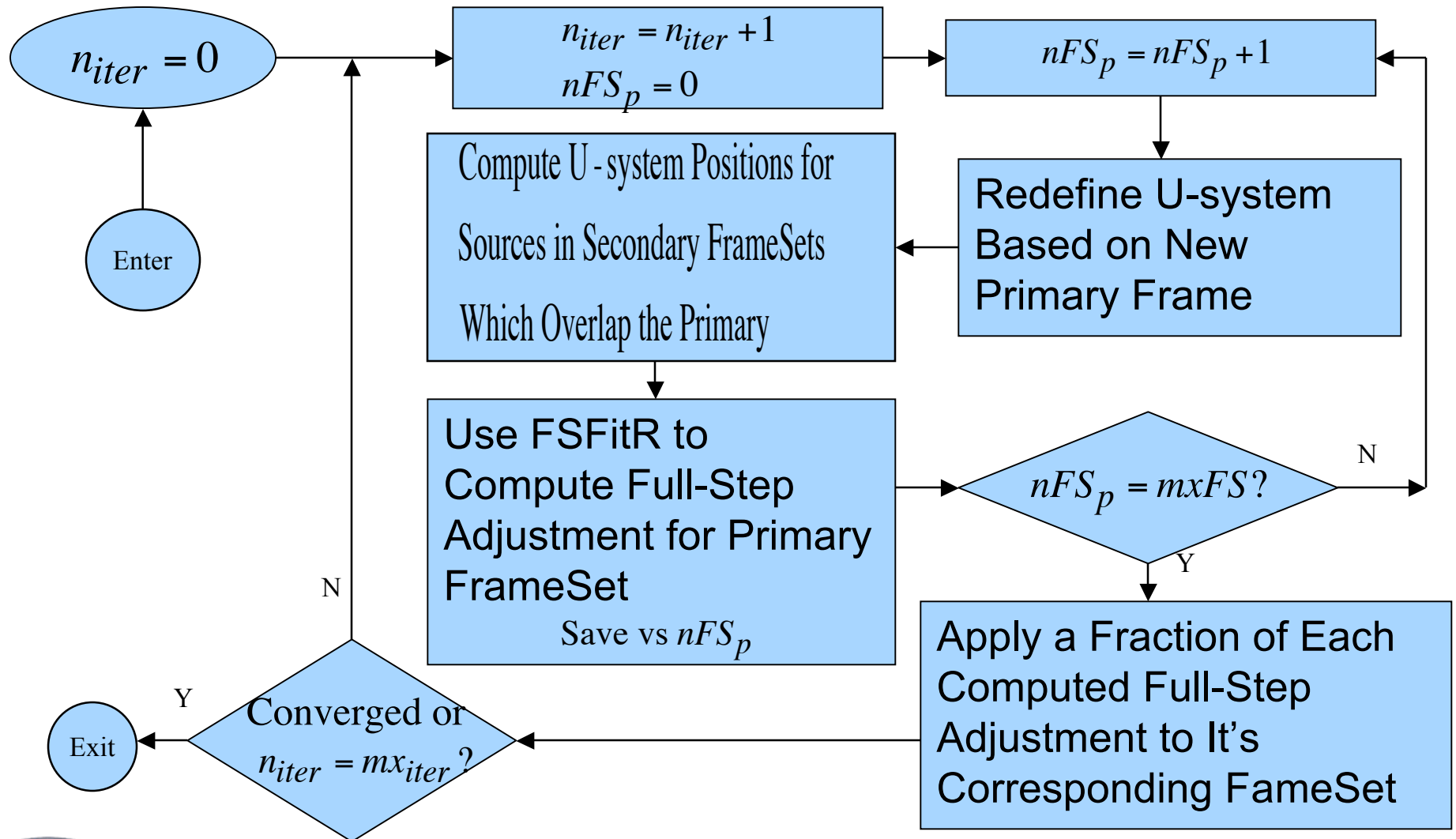




Functional MFPRex FlowChart



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CPU Time Sensitive MFPReX Design Issues



- Use of FSFitR for primary frameset fitter
 - Provides full flexibility (any subset of the 20 parameters)
 - Could recover low source count W3 & W4 frames
 - Already developed & tested in SFPReX
- Redefinition of U-system for each frameset
 - Allows unlimited footprint size (up to 4π steradians)
 - Cartesian system never covers more than ~ 1 degree





PRex Development Schedule



- May 01, 2008: All liens removed from SFPReX (except proper motions)
MFPReX design complete
(issues dependent on CPU timing tests remain)
- June 01, 2008: 2MASS proper motion modeling available
- July 17, 2008 (WSDS v1): “Complete” SFPReX code delivered
“Prototype” MFPReX code delivered
- Sept 15, 2008: Decision on CPU time sensitive MFPReX design issues
- Dec 01, 2008: “Preliminary” MFPReX code delivered
- Feb 28, 2009 (WSDS v2): "Mature" SFPReX code delivered
“Complete” MFPReX code delivered
- Aug 04, 2009 (WSDS v3): “Mature” MFPReX code delivered





Liens/Concerns



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- The 20-parameter fit routine (FSFitR) not yet installed
- Outlier detection and rejection algorithms have not yet been designed.
- SIS's have not yet been written for output files.
- Refinement of pattern match efficiency & acceptance criteria
- How to handle lack of proper motions in 2MASS undecided
- Uncertainties coming out of pattern matcher (*apriori* inputs to FSPRex)
- Prediction of which 2MASS stars likely to show up in W1, W2, W3 & W4
- Predictions of % W1 sources also found in W2, W3 & W4;
 % W2 sources also found in W3 & W4; % W3 sources also in W4
- Recourse if bands one and two are both missing
- Verification via testing of CPU time sensitive MFPREX design issues
 - Use of FSPRex for primary frameset fitter
 - Redefinition of U-system for each frameset
- Fallback position if MFPREX CPU time test results unfavorable

