Wide-field Infrared Survey Explorer (WISE)

GnDSTR Subsystem Design Document

Version 1.0

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Revision History

Date	Version	Author	Description
1 October 2008	1.0	J. W. Fowler	Initial Draft
2 October 2008	1.0	J. W. Fowler	Corrections to some notation

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1 Introduction

1.1 Subsystem Overview

This document presents the requirements, design, algorithms, and state of implementation of the GnDSTR (Generate Distortion) subsystem of the WSDC data processing system. GnDSTR runs offline on a list of RvB files generated by the sfprex module of the PRex subsystem.

1.1.1 Requirements

The GnDSTR subsystem is required to compute the coefficients of a polynomial distortion model by performing a chi-square minimization fit using a collection of matched pairs of point sources, where each pair consists of one WISE source and one 2MASS source. Both forward and backward models are required, i.e., conversions from distorted coordinates to undistorted coordinates and vice versa. Other requirements include optional output files containing the error covariance information for the polynomial coefficients, a table file of residuals, and an image of the distribution of residual magnitudes over the imaging array. The Level 4 requirements supported by this processing are as follows.

> L4WSDC-014: The root mean square (1σ) error in WISE catalog positions with respect to 2MASS All-Sky Point Source Catalog positions shall be less than 0.5" on each axis, for sources with SNR > 20 in at least one WISE band.

> L4WSDC-017: The WISE Source Catalog shall contain equatorial (J2000) coordinates for objects detected in at least one band.

> L4WSDC-018: The WISE Source Catalog shall contain uncertainties in the coordinate measurements for each object.

> L4WSDC-046: The WSDS Pipeline processing shall reconstruct the J2000 equatorial positions of sources detected on the calibrated WISE images relative to the positions of objects in the 2MASS All-Sky Point Source Catalog that are detected in the WISE science images.

> L4WSDC-062: The WSDC shall perform quality analysis of all WISE science data and make reports available on a regular basis.

> L4WSDC-078: The WISE science data products shall use the International Celestial Reference System (ICRS) to describe the positions and motions of celestial bodies. WISE astrometry shall be mapped into the ICRS using the 2MASS All-Sky Point Source Catalog as the primary astrometric reference.

1.1.2 Liens

- 3rd-order and 5th-order polynomial processing has not yet been implemented.
- Error analysis for the skew parameter has not been performed formally; Monte Carlo studies have shown that its error is of the same order as the other parameters of the model, but since it is obtained iteratively and expected to be negligibly small given a reasonable amount of data to fit, a formal expression is currently considered not feasible.
- SIS's have not yet been written for any output files.

1.2 Applicable Documents

This subsystem conforms to the specifications in the following project documents:

- WISE Science Data Center Functional Requirements Document, WSDC D-R001
- WISE Science Data System Functional Design, WSDC D-D001
- Software Management Plan, WSDC D-M003
- PRex SDS, WSDC D-D003
- SIS describing the WCS and distortion representation
  http://web.ipac.caltech.edu/staff/fmasci/home/wise/SFPWrap01.txt
- Formalism deriving the representation and how it is coupled to the CD matrix
  http://web.ipac.caltech.edu/staff/fmasci/home/wise/codeVdist.html
- RvB SIS, WSDC D-I117
  http://web.ipac.caltech.edu/staff/roc/wise/docs/sis/PRex_RvB.SIS

1.3 Acronyms

2MASS	Two-Micron All-Sky Survey
FRD	Functional Requirements Document
GnDSTR	Generate Distortion Model
PRex	Position Reconstruction
SDS	Subsystem Design Specification
SIS	Software Interface Specification
W1	WISE wavelength channel 1, 3.3 microns
W2	WISE wavelength channel 2, 4.7 microns
W3	WISE wavelength channel 3, 12 microns
W4	WISE wavelength channel 4, 23 microns
WISE	Wide-field Infrared Survey Explorer
WSDC	WISE Science Data Center
WSDS	WISE Science Data System

2 Input

2.1 Control Input

GnDSTR reads control input exclusively in the form of command-line parameters which are shown in its tutorial display:

```
gnDSTR: Generate Distortion Model vsn 1.22 A80624

 usage:  gndstr <flags> <specifications>

 where <flags> <specifications> must be:

    -l   ListFile   (Name of file containing a list of RvB
                     input file names; required)
    -o   OutFile    (Coefficient output file name; required)
    -u   UncFile    (Uncertainty output file name; optional)
    -r   ResidFile  (Residuals output file name; optional)
    -rf  ResidFITS  (Residuals output FITS file name; optional)
    -nf  NFITS      (No. rows/columns in FITS file; optional;
                     default = 31)
    -p   PolyOrd    (Polynomial order, 3, 4, or 5; optional;
                     default = 4)
    -m   MaxIter    (Maximum no. iterations; optional;
                     default = 1000)
    -t   Tolerance  (Convergence criterion for relative change
                     in beta; optional; default = 1.0d-9)
    -c   ChSqMax    (Maximum 2-D chi-square to keep data point
                     in average; optional; default = 8.0d0)
    -d              (Debug output; optional; default = F)
```

2.2 Source Detections and Astrometric Reference Input

GnDSTR reads the list of RvB files contained in the "ListFile" specified via the "-l" command-line parameter. This file contains one file name per line, left-justified, case-sensitive, and containing any needed path information. The RvB files (see the SIS identified in section 1.2) contain matched pairs of 2MASS and WISE detections; the latter have positions that may or may not be already corrected by a current distortion model. In the former case, what follows is a *differential* correction to that model; the description will be that of a full distortion correction.

3   Processing

3.1 Initialization

The GnDSTR module initializes itself by:

A.) reading and processing its control inputs;

B.) allocating memory for arrays needed for residuals analysis;

C.) setting up table-file names for case-insensitive I/O;

D.) initializing summations used in the chi-square minimization;

E.) opening the list file for I/O.

3.2 Polynomial Fitting

The GnDSTR module solves for the polynomial coefficients of a distortion model via chi-square minimization. The input data are positions and uncertainties of the matched pairs of WISE and 2MASS point sources read from the RvB files. The list of such files is read as though all the files were concatenated into one input source list which is read from top to bottom each time a pass through the iterative solution is performed. In practice, each RvB is separately opened and processed for table-file column information, but this is transparent to the fitting procedure, which sees only one stream of source data per iteration.

The form of the distortion model depends on the polynomial order, N. The two axes each have their own polynomial, and these are coupled later through the skew parameter. Forward and backward models are both computed; these are indicated as F2S (Frame-to-Sky) and S2F (Sky-to-Frame) models, i.e., conversion from distorted coordinates to undistorted coordinates and vice versa, respectively. Thus there are four equations whose coefficients must be computed, two F2S equations covering the "frame" X and Y axes, and two S2F equations covering the "sky" X and Y axes. The "sky" and "frame" coordinates are Cartesian systems corresponding to the same tangent projection but differing only in whether distortion operates. Each of the four equations has the same form but different coefficients. For illustration, we will consider only the F2S equation for X; the distortion on the frame X axis is denoted du.

$$du = \sum_{n=0}^{N}\sum_{k=0}^{n} a_{nk} X^{n-k} Y^{k} \tag{1}$$

The coefficients are bookkept as single-index arrays in the code; for example, the 4th-order polynomial model has 15 coefficients per axis per conversion, `A1(j)` for F2S and `A2(j)` for

S2F, with j running from 1 to 15 for *X* and from 16 to 30 for *Y*, with a mapping between j and the powers of *X* and *Y* for *du* given by:

j	n-k	k
1	0	0
2	1	0
3	2	0
4	3	0
5	4	0
6	0	1
7	1	1
8	2	1
9	3	1
10	0	2
11	1	2
12	2	2
13	0	3
14	1	3
15	0	4

The distortion on the *Y* axis is denoted *dv*, the polynomial has the same form, but the coefficients in the polynomial corresponding to Equation 1 are denoted b_{nk}, and the coefficient index mapping is given by adding 15 to each of the j values above. The S2F conversion uses a similar mapping for the A2 array. The 3rd-order polynomial (not yet implemented) has 10 coefficients per axis per conversion, and the 5th-order polynomial (not yet implemented) has 21.

Since *X* and *Y* can refer to axes in either sky or frame coordinates and to WISE detections or 2MASS sources, the following notation is used to keep the various combinations distinct:

u: *X* position of extraction in band frame (pixels)
v: *Y* position of extraction in band frame (pixels)
du: distortion adjustment along *X* axis (pixels)
dv: distortion adjustment along *Y* axis (pixels)
xr: reference star undistorted position in band frame along *X* axis (pixels)
yr: reference star undistorted position in band frame along *Y* axis (pixels)
xm: distortion-corrected WISE star position in band frame along *X* axis (pixels)
ym: distortion-corrected WISE star position in band frame along *Y* axis (pixels)
β: skew angle, assumed to be small (rad)

The (*xr, yr*) coordinates are obtained by transforming the 2MASS celestial coordinates into band-frame pixel coordinates assuming no distortion exists, i.e., transforming according to frame center offsets, frame rotation, and scale factors only. If there were indeed no distortion, then these would be the expected coordinates for the matching WISE detection, (*u, v*). Correcting the latter for distortion yields (*xm, ym*), which then *are* the expected 2MASS coordinates. The

distortion correction has the following form, where skew is arbitrarily assigned to the Y axis (either axis can be used as long as skew is treated consistently throughout).

$$xm = u + du$$
$$ym = -\beta(u + du) + (v + dv) \tag{2}$$

Because of source position errors in both 2MASS and WISE, (xr, yr) and (xm, ym) are generally not equal for each given matched pair of 2MASS/WISE sources. The a_{nk} coefficients are determined via chi-square minimization of this discrepancy. For each matched pair, the position error covariance matrices are added to obtain the expected error covariance matrix for the difference, and this supplies the weights used in the chi-square minimization.

$$\begin{pmatrix} V_x & V_{xy} \\ V_{xy} & V_y \end{pmatrix} = \begin{pmatrix} V_x & V_{xy} \\ V_{xy} & V_y \end{pmatrix}_{2MASS} + \begin{pmatrix} V_x & V_{xy} \\ V_{xy} & V_y \end{pmatrix}_{WISE}$$
$$\begin{pmatrix} W_x & W_{xy} \\ W_{xy} & W_y \end{pmatrix} = \begin{pmatrix} V_x & V_{xy} \\ V_{xy} & V_y \end{pmatrix}^{-1} \tag{3}$$

The chi-square to be minimized is

$$\chi^2 = \chi_x^2 + \chi_{xy}^2 + \chi_y^2$$
$$\chi_x^2 \equiv \sum W_x (xm - xr)^2$$
$$\chi_{xy}^2 \equiv 2 \sum W_{xy} (xm - xr)(ym - yr) \tag{4}$$
$$\chi_y^2 \equiv \sum W_y (ym - yr)^2$$

where the summations are taken over all matched pairs of 2MASS and WISE sources in the RvB input. The derivatives of chi-square with respect to du, dv, and β are taken formally, using Maple. Since du and dv are explicit functions of the a_{nk} and b_{nk} (as seen in Equation 1 for du), derivatives of chi-square are actually taken with respect to the a_{nk} and b_{nk}, and so the number of equations obtained is equal to the number of coefficients. This forms a linear system of equations for the a_{nk} and b_{nk} given any value of the skew. Including the skew derivative equation would result in a nonlinear system, and so instead, skew is obtained iteratively by assuming a value to be used in the linear system, which can be solved without iteration. Once the a_{nk} and b_{nk} have been obtained, they can be used in the skew derivative equation, which is set to zero, and a solution for β is obtained. This can be used in another solution of the linear system, after which another value for β is obtained, and this process can be repeated until the β value converges. In practice, it is found that the solution oscillates, and so a damped convergence is used, i.e., each iteration uses a value for β that is the average of the solutions for the *two* previous iterations. This has been found to converge to better than single precision in only a few iterations (for the 4th-order polynomial applied to a Code V model of the distortion). The skew is assumed to be zero for the first iteration.

After the first iteration hase been performed, subsquent iterations can employ the estimates of the previous iteration to compute a chi-square contribution for each source pair, i.e., Equation 4 without the summation symbols. This yields a chi-square value with two degrees of freedom; this value must be less than the threshold `ChSqMax` (see "-c" in section 2.1). The default value is 8, which implies that the expected fraction of the population rejected as outliers is 1.83%. The number of data pairs retained and the overall reduced chi-square values for the F2S and S2F models are all computed and printed in the processing summary.

The equations making up the linear system are extremely unwieldy and will not be duplicated herein; the Maple worksheets and Fortran code are available upon request.

3.3 Uncertainties

As described in section 3.2 above, the a_{nk} and b_{nk} are evaluated via a standard chi-square minimization procedure assuming a current estimate of β. The linear system can be expressed in matrix form, where the matrix elements are the coefficients of the a_{nk} and b_{nk}. The error covariance matrix for the estimated values of the a_{nk} and b_{nk} is the inverse of the coefficient matrix. Tests with Code V data showed what is normally to be expected, that the errors in the a_{nk} and b_{nk} estimates are strongly correlated. This means that a realistic estimate of the uncertainty of the distortion model needs to take the entire error covariance matrix into account.

This is usually not a problem, but in this case, the solutions for the a_{nk} and b_{nk} are just one step in an iterative procedure that includes solving for the skew angle, β. The error in β is therefore correlated to those of the a_{nk} and b_{nk}, but the functional dependence is made extremely complicated by the iterative procedure. The actual errors in the distortion correction are expected to be small, and it is reasonable to expect that they may possibly be made negligible. For this reason, it is not cost effective to invest the large amount of resources in developing a formal error model that such a task would require. It was decided instead to develop an approximate error model via Monte Carlo analysis. This is described in section 5.2 below, where it is seen that the number of 2MASS/WISE matched sources needed to drive the expected error into negligibility is well within the expected size of such data.

4   Output

4.1 Model Parameters Output File

An output table file of coefficients for the F2S and S2F distortion models is produced. the following is an example from a run using Code V data.

```
\ gnDSTR vsn 1.23 A80911 run on 23-09-08 at 10:44:57
\beta_F2S   =  6.53766764E-09
\beta_S2F   = -6.33628835E-09
\Num_iter   =             15
\Poly_ord   =              4
\Tol        =      1.000E-09
\Num_points =           2883
\Num_used   =           2503
\ChSqMax    =      8.000E+00
\ChSqF2S    =      1.442E+00
\ChSqS2F    =      1.443E+00
| Cn| C |     Ai_F2S     |     Ai_S2F     |
| i | c |     real       |     real       |
   1 A11  1.33116529E-05 -1.31887706E-05
   2 A12 -1.42229420E-07  1.42618115E-07
   3 A13  7.20222047E-11 -7.23060961E-11
   4 A14  2.23501979E-13 -2.25074489E-13
   5 A15 -8.65382841E-16  8.61168975E-16
   6 A21 -6.42685096E-04  6.46270984E-04
   7 A22 -4.16811442E-06  4.16870242E-06
   8 A23 -9.24317237E-12 -1.27979160E-11
   9 A24 -1.21079032E-12  1.18906102E-12
  10 A31 -1.11108016E-10  1.10138245E-10
  11 A32  4.13450098E-13 -4.15677696E-13
  12 A33  5.64735140E-16 -5.51788707E-16
  13 A41  3.58534022E-09 -3.61333211E-09
  14 A42  3.45699983E-12 -3.41888096E-12
  15 A51  3.60851661E-18 -4.16869157E-18
  16 B11  1.16207817E+00 -1.16259915E+00
  17 B12  4.74842083E-04 -4.71178578E-04
  18 B13 -2.92214918E-06  2.93712000E-06
  19 B14 -2.74088961E-09  2.74091897E-09
  20 B15 -9.27014334E-13  8.79645106E-13
  21 B21  1.07071454E-08 -1.07717902E-08
  22 B22 -1.38953839E-10  1.36623699E-10
  23 B23  3.13401460E-14 -3.13127045E-14
  24 B24  3.29727840E-16 -3.23232256E-16
  25 B31 -9.04694997E-06  9.03926018E-06
  26 B32 -4.06717274E-10  3.03056906E-10
  27 B33  4.08381476E-12 -4.16301721E-12
  28 B41 -7.76028201E-14  7.64986264E-14
  29 B42  3.30987370E-16 -3.27928983E-16
  30 B51 -1.29406944E-12  1.35728574E-12
```

4.2 Polynomial Parameter Error Covariance Matrix Output File

An outout table file containing the values of the error covariance matrix for the a_{nk} estimates can be obtained via the "-u" command-line option (see section 2.1). An example is shown below.

```
\ gnDSTR vsn 1.23 A80911 run on 23-09-08 at 10:44:57
| i| j| Sig_CoSig_F2S_Aij | Sig_CoSig_S2F_Aij | Var_CoVar_F2S_Aij | Var_CoVar_S2F_Aij | Corr_F2S_Aij | Corr_S2F_Aij|
| i| i|       real        |       real        |       real        |       real        |     real     |     real    |
  1  1   1.88493890797E-01   1.88483408749E-01   3.55299468676E-02   3.55259953735E-02    1.000000     1.000000
  2  1  -5.23945466218E-04   5.94918485182E-04  -2.74518851571E-07   3.53928004011E-07   -0.002246     0.002897
  3  1  -6.21503608646E-04  -6.21213313204E-04  -3.86266735560E-07  -3.85905980502E-07   -0.633605    -0.633429
  4  1   1.31058955415E-06  -1.00877273733E-06   1.71764497944E-12  -1.01762243559E-12    0.002731    -0.001619
  5  1   1.01308699505E-06   1.01220802000E-06   1.02634525955E-12   1.02456507575E-12    0.431332     0.431100
  6  1   9.84556496914E-07   1.44700621462E-06   9.69351495615E-13   2.09382698516E-12    0.000000     0.000000
  7  1  -1.21360014632E-07  -7.72847118509E-08  -1.47282531514E-14  -5.97292668587E-15   -0.000000    -0.000000
  8  1  -6.33277816985E-10   2.57367500775E-09  -4.01040793485E-19   6.62380304554E-18   -0.000000     0.000000
  9  1   2.17973665402E-10   1.64865481975E-10   4.75125188088E-20   2.71806271468E-20    0.000000     0.000000
 10  1  -6.21243908130E-04  -6.21416042843E-04  -3.85943993389E-07  -3.86157898302E-07   -0.632531    -0.632647
  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
 21 30   1.40665369636E-11   2.41527542122E-11   1.97867462149E-22   5.83355536035E-22    0.000000     0.000000
 22 30  -1.31119621065E-12  -6.56561151993E-13  -1.71923550283E-24  -4.31072546307E-25   -0.000000    -0.000000
 23 30  -2.47142270018E-14   4.46648951354E-14  -6.10793016299E-28   1.99495285745E-27   -0.000000     0.000000
 24 30   2.08200780710E-15   2.66761707706E-15   4.33475650882E-30   7.11618086985E-30    0.000000     0.000000
 25 30  -6.15296197658E-09  -6.15720398202E-09  -3.78589410853E-17  -3.79111608762E-17   -0.922070    -0.922093
 26 30   3.08026617678E-12  -1.88512473826E-11   9.48803971983E-24  -3.55369527879E-22    0.000220    -0.008229
 27 30   3.78354053229E-12   3.78814316015E-12   1.43151789555E-23   1.43500286018E-23    0.093679     0.093797
 28 30  -1.15966081343E-14  -7.60704345447E-14  -1.34481320221E-28  -5.78671101182E-27   -0.000000    -0.000000
 29 30   1.63842509109E-15  -2.96230759208E-15   2.68443677911E-30  -8.77526627008E-30    0.000000    -0.000000
 30 30   1.26841020667E-11   1.26958498928E-11   1.60886445237E-22   1.61184604500E-22    1.000000     1.000000
```

This example is from a file containing 900 data rows for the 30×30 error covariance matrix corresponding to the 4th-order polynomial. The i and j columns give the coefficient number between 1 and 30 according to the mapping shown in section 3.2 above. For simplicity, no attempt is made to take of advantage of the matrix symmetry. The next two columns contain the sigma or co-sigma, as appropriate, for the coefficient(s) corresponding to the i and j combination for the F2s and S2F models, respectively, and the next two columns give the corresponding variances or covariances, respectively. The last two columns give the corresponding correlation coefficients, respectively.

4.3 Residuals Statistics Output File

An optional output table file of residuals to the model fit may be generated via the "-r" command-line specification (see section 2.1). An example is shown below.

```
\ gnDSTR vsn 1.0  A80604 run on 12-06-08 at 11:52:14
| Nused |    xfe    |    yfe    |  xdstr  |  ydstr  |   du    |   dv    |  xresd  |  yresd  |
|  int  |   real    |   real    |  real   |  real   |  real   |  real   |  real   |  real   |
      1   -512.7442   -514.0885   -1.1936   -1.8819   -1.1941   -1.8837   -0.0005   -0.0018
      2   -478.5083   -513.7827   -1.0614   -1.5761   -1.0616   -1.5773   -0.0002   -0.0012
      3   -444.2832   -513.4972   -0.9391   -1.2906   -0.9400   -1.2918   -0.0009   -0.0012
      4   -410.0684   -513.2324   -0.8279   -1.0258   -0.8286   -1.0272   -0.0007   -0.0014
      5   -375.8631   -512.9883   -0.7263   -0.7817   -0.7266   -0.7836   -0.0003   -0.0019
      6   -341.6662   -512.7667   -0.6322   -0.5601   -0.6332   -0.5609   -0.0010   -0.0008
      7   -307.4773   -512.5649   -0.5470   -0.3583   -0.5476   -0.3594   -0.0006   -0.0011
      8   -273.2951   -512.3848   -0.4686   -0.1782   -0.4690   -0.1789   -0.0004   -0.0007
      9   -239.1199   -512.2253   -0.3959   -0.0187   -0.3967   -0.0196   -0.0008   -0.0009
     10   -204.9503   -512.0875   -0.3301    0.1191   -0.3299    0.1185    0.0002   -0.0006
     11   -170.7844   -511.9708   -0.2680    0.2358   -0.2678    0.2355    0.0002   -0.0003
     ...............................................................................................
```

This lists, for each source pair retained in the fit, its sequence number among retained pairs in the input stream, `NUsed`, the extraction coordinates in the band-frame system (`xfe, yfe`), the amount of distortion on each axis deduced by subtracting the 2MASS coordinates from the WISE coordinates (`xdstr, ydstr`), the corresponding model values (`du, dv`), and the residual differences between the observed and model distortion (`xresd, yresd`). Normally the latter would be typical of the position errors in the 2MASS and WISE sources, but in this example the individual residuals are much smaller because the data were simulated from a Code V model of the optical system, and hence here the individual residuals are a direct measure of the model accuracy. The X and Y residuals never reached 0.002 pixel in magnitude.

4.4 Residuals Image Output Files

A set of FITS images showing the distributions of various residual parameters as functions of array location is available via the "-rf" command-line option (see section 2.1). There are 18 images in the set, each by default a 31×31 image, i.e., the array is partitioned into 31 rows and 31 columns, and the residual parameters are averaged within each partition. The number of rows and columns may be controlled via the "-nf" command-line parameter. Each image has a suffix attached to the name specified by the user; the meanings are as follows, where "Img" represents the name specified by the user.

|	
Img_XF2S.fits:	X of distortion-corrected WISE – 2MASS, F2S model
Img_YF2S.fits:	Y of distortion-corrected WISE – 2MASS, F2S model
Img_RF2S.fits:	Radial distance from distortion-corrected WISE to 2MASS, F2S model
Img_XS2F.fits:	X of distortion-corrected WISE – 2MASS, S2F model
Img_YS2F.fits:	Y of distortion-corrected WISE – 2MASS, S2F model
Img_RS2F.fits:	Radial distance from distortion-corrected WISE to 2MASS, S2F model
Img_XD.fits:	X distortion, extracted WISE – 2MASS
Img_YD.fits:	Y distortion, extracted WISE – 2MASS
Img_RD.fits:	Radial distortion, extracted WISE – 2MASS
Img_XF2Ssig.fits:	Dispersion about XF2S partition averages
Img_YF2Ssig.fits:	Dispersion about YF2S partition averages
Img_RF2Ssig.fits:	Dispersion about RF2S partition averages
Img_XS2Fsig.fits:	Dispersion about XS2F partition averages
Img_YS2Fsig.fits:	Dispersion about YS2F partition averages
Img_RS2Fsig.fits:	Dispersion about RX2F partition averages
Img_XDsig.fits:	Dispersion about XD partition averages
Img_YDsig.fits:	Dispersion about YD partition averages
Img_RDsig.fits:	Dispersion about RD partition averages
Img_N.fits:	Number of points averaged in partition

Some sample images from the Code V data are:
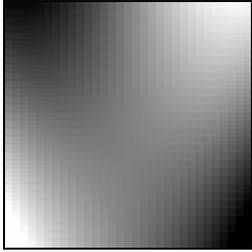
XD: X distortion
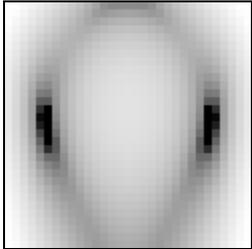
RD: Radial distortion
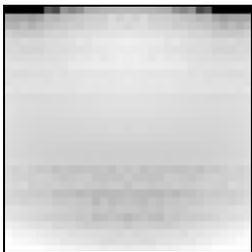
YS2F: Y residuals for S2F model

5   Testing

5.1 Testing

Testing was performed with data generated for W1 with Code V. The data consisted of 961 ray traces at each of three scan-mirror position, center and both extremes. These scan-mirror positions were averaged and reformatted to look like 961 WISE/2MASS matched source pairs. There was no position error simulated for these sources, and so the residuals indicate the adequacy of the 4th-order polynomial for the distortion modeled by Code V. These residuals were always smaller in magnitude that 0.002 pixel on both axes. The distortion pattern for the averaged mirror position as a function of array location is shown in Figure 1 below, where the distortion dispacements are magnified relative to the plate scale by a factor of 15, and the base points are the dots. The most extreme displacements are about 2 pixels.
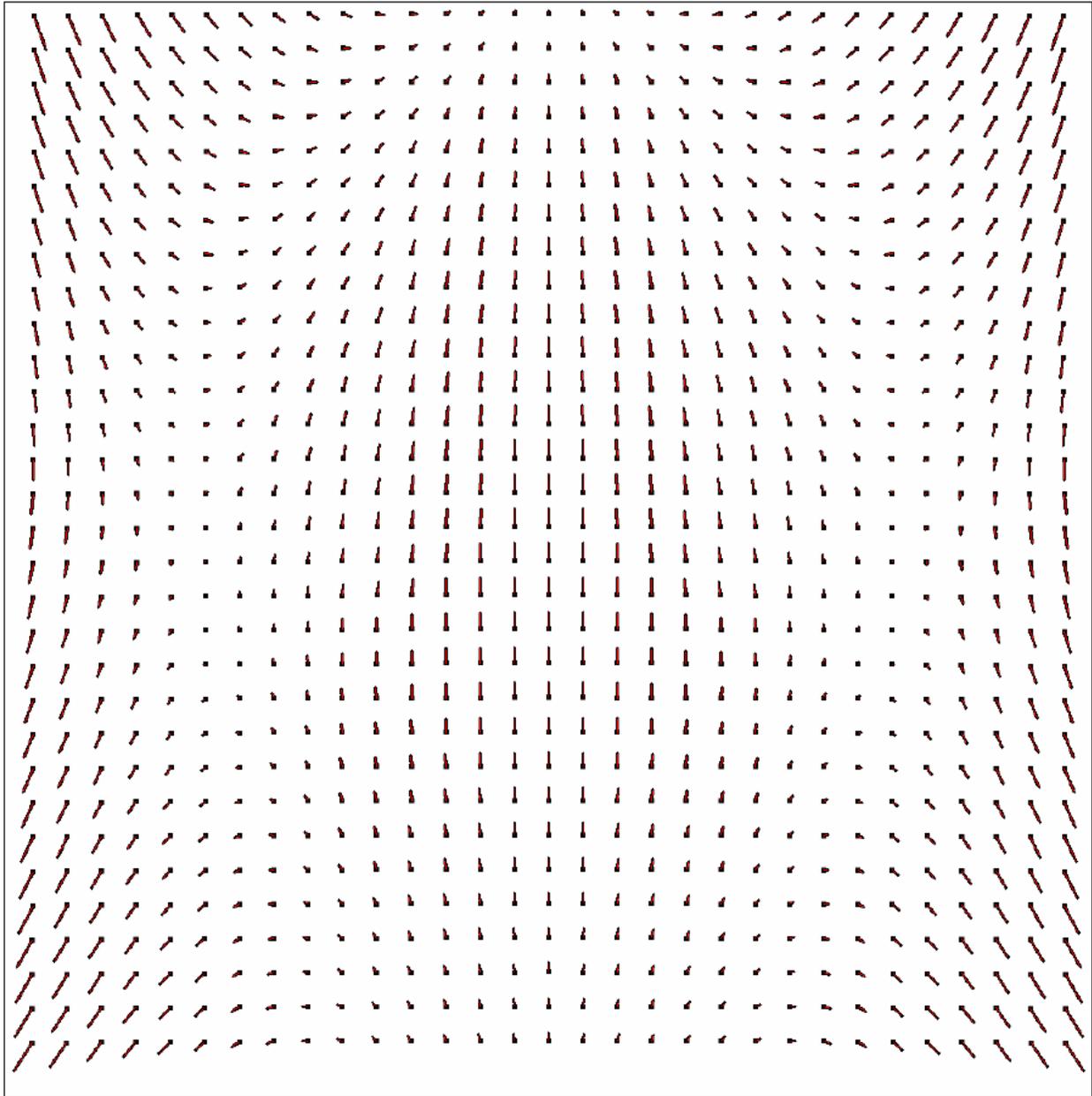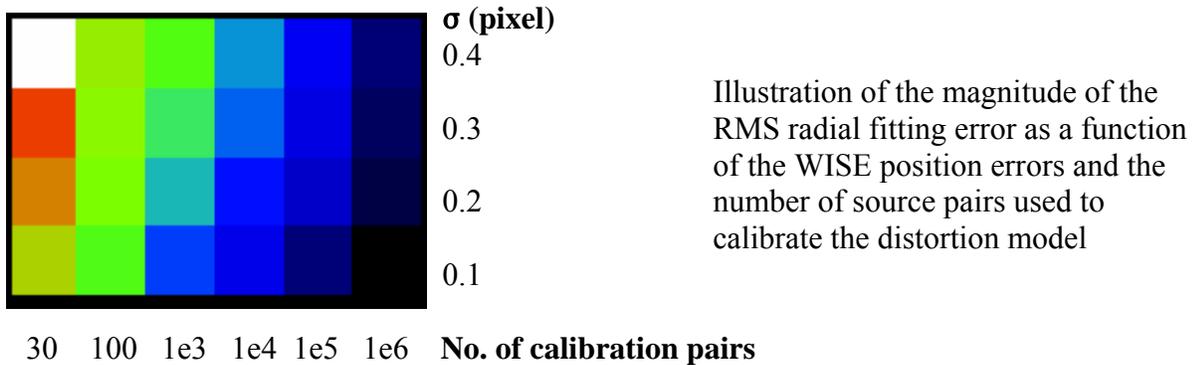
Figure 1. Distortion offset pattern as a function of array location (W1, Code V, averaged mirror position); undistorted positions are shown as dots, and displacements are magnified by a factor of 15.

5.2 Monte Carlo Analysis

Monte Carlo analysis was performed for the purpose of exploring the possibility of developing an approximate error model to replace the formal error model, because development of the latter would clearly not have been cost-effective given the extreme complications due to the partly decoupled iterative solution and the anticipated small error size. The Monte Carlo program generated a requested number of simulated WISE/2MASS source pairs in RvB format. Distortion could be added or not added to the WISE positions; if added, the 4th-order polynomial distortion model with skew was assumed, and the coefficients could be read in or defaulted to the Code V model for the averaged scan-mirror position. Running without distortion was used to verify proper simulation of position errors and to probe fitting distortion despite its absence.

2MASS position errors were taken to be 0.03 pixel on each axis. Multiple simulations were done for 30, 100, 1000, 10000, 100000, and 1000000 source pairs and 0.1, 0.2 and 0.4 pixel 1-sigma WISE position errors (0.3 pixel errors were interpolated for the tables below). Distortion correction errors were computed and averaged over the array for null true distortion and again for Code V distortion.

σ (pixel)
0.4
0.3
0.2
0.1

30   100   1e3   1e4   1e5   1e6   **No. of calibration pairs**

Illustration of the magnitude of the RMS radial fitting error as a function of the WISE position errors and the number of source pairs used to calibrate the distortion model

The following tables are gridded according to the same WISE position errors and number of source pairs as the above graphical representation of RMS radial fitting-error magnitude. They show the radial error of the distortion correction averaged over the array for the model computed for the corresponding number of source pairs and WISE position uncertainties. The radial error is computed by subtracting the correction given by the estimated model from that of the true model.

1-σ Radial Error in Distortion Correction Averaged Over the Array (Pixel; Null Distortion)

0.71090	0.18036	0.07090	0.02613	0.00781	0.00194
0.55784	0.15201	0.05212	0.01960	0.00612	0.00156
0.40479	0.12366	0.03334	0.01307	0.00443	0.00119
0.24049	0.07010	0.01636	0.00670	0.00199	0.00042

1-σ Radial Error in Distortion Correction Averaged Over the Array (Pixel; Code V Distortion)

1.53083	0.33140	0.06918	0.02099	0.00640	0.00288
1.02270	0.23142	0.05946	0.01596	0.00503	0.00213
0.51458	0.13144	0.04974	0.01094	0.00367	0.00138
0.35327	0.07586	0.01655	0.00500	0.00192	0.00078

For small numbers of source pairs, inaccurate models are obtained, even when there is no distortion simulated; these produce errors that dominate the source position errors. When more source pairs are used, the model becomes more accurate. For at least 100,000 source pairs, the model becomes about as accurate as that derived from Code V with no source position error included, i.e., on the order of 0.002 pixel. This amount of data shoud be easily attainable early in the mission, and therefore the relatively expensive bookkeeping of the error contributed by the distortion model is taken to be not cost effective; this error will be considered negligible compared to the other effects contributing to position error.

The values in each grid of the tables above are actually averages of three Monte Carlo runs made with the corresponding position errors and number of source pairs (except for the row for 0.3 pixel position error, which was later interpolated).