Report Title WISE Band 1 Image Quality Post-Vibe	Document Number SDL/09-308
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1. INTRODUCTION

WISE image quality is measured in noise pixels (N_p), defined as:

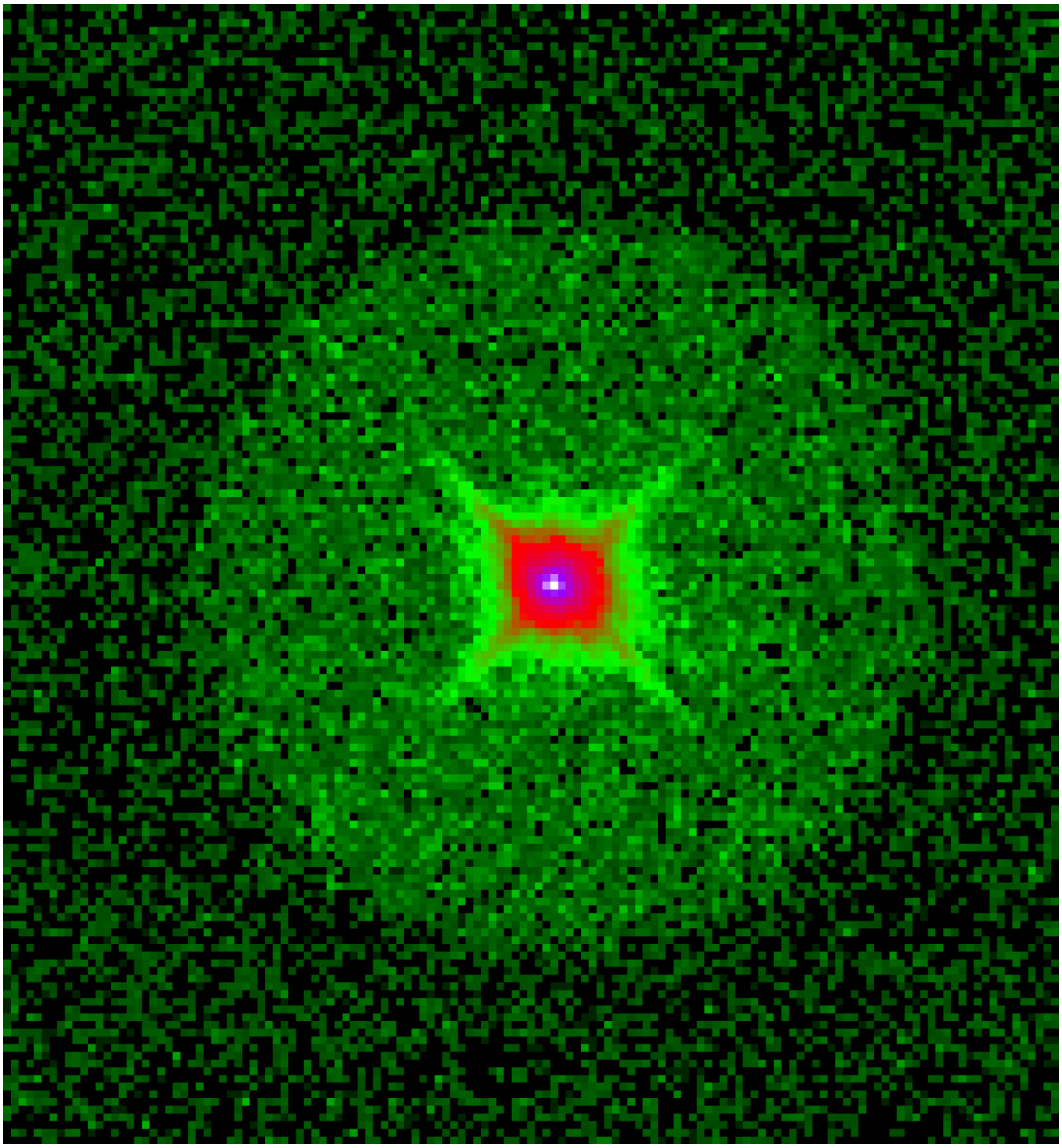
$$N_p = \frac{\left(\sum V_{i,j}\right)^2}{\sum V_{i,j}^2}$$

$V_{i,j}$ is the value on a pixel and the sums are over all pixels containing power from a point source. It is a measure of how many pixels worth of noise are present in a measurement of source irradiance if a PRF is fitted to a point source.

In principle, noise pixels should be simple to measure from any image of a point source, but it is not simple in practice.

- A background value is generally present and must be subtracted out, even in an image which has had a background image subtracted
- The background must be measured over an area large enough to avoid pixel noise in the background value but small enough that the value is the same as that under the source
- The noise pixels must be measured over a large enough area to include all the significant power from the point source
- The noise pixels must be measured over an area small enough that pixel noise does not dominate the result.

At the minimum, there is a background due to structure in the test setup and due to variation in the background over an array readout column (at a few count level, this can be significant in some cases even when peak source signal is ~4000 counts). The background from the test setup is the far greater effect and is due to the aperture holder heating up under blackbody illumination and emitting at a low but significant level. The aperture holder is seen in the image below as the 46-pixel radius circle around the source. The aperture holder consists of two sections, an outer annulus and an inner circle (of 20 pixel radius to WISE), which are different materials and thicknesses and could emit at different levels. However, the image below and radial plots of this data shows this is not the case and any difference in radiance from these sections is insignificant.



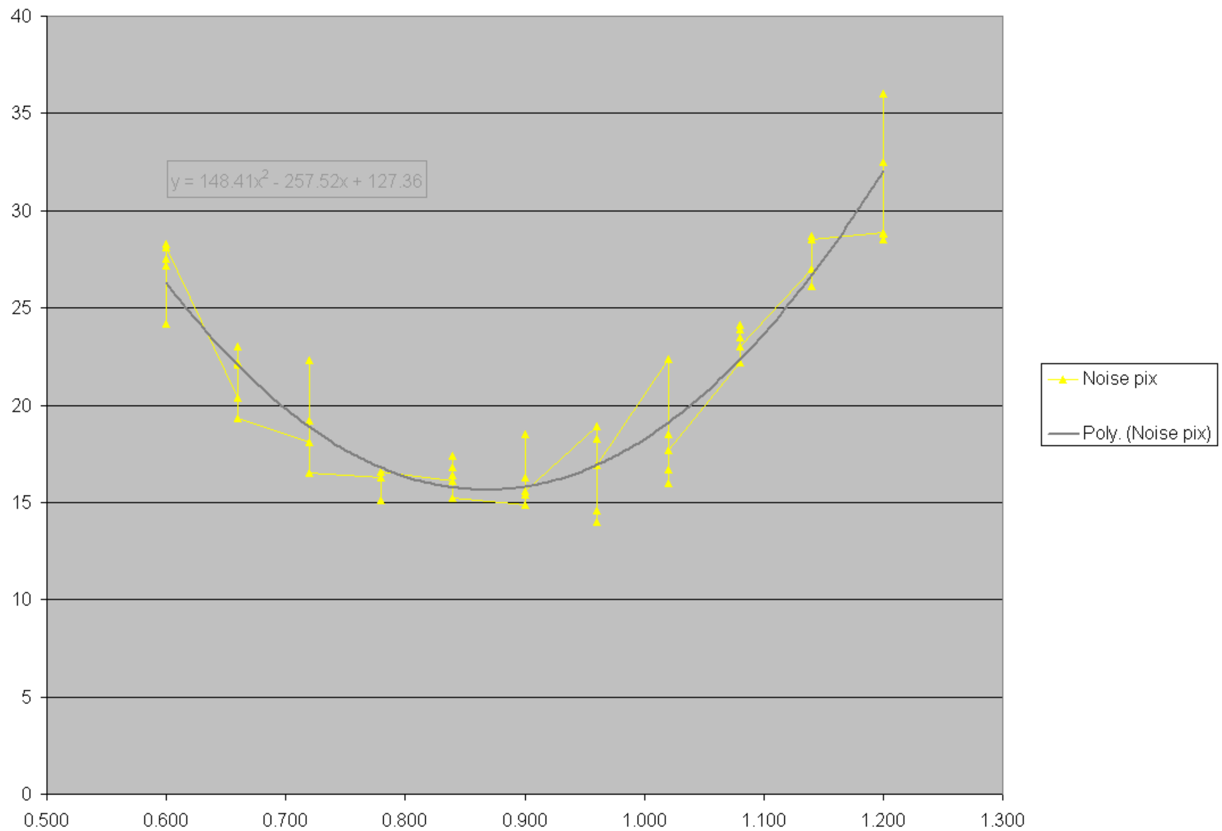
The above image is a composite of several images of near angle scatter data from the 3rd blue tube test when the same source was imaged at several filtering levels and integration times. The image is on a logarithmic scale and the color scale used emphasizes low levels.

The best way to determine the area where noise pixels should be counted and background measured (though obviously this must be done within 46 pixels of the source) is to look at plots of the sum of all pixels inside a given radius vs. radius for several images of the source. For a constant source, as used in all the focus data collection, the total signal from the source will remain constant. These plots make clear:

- The presence of a background and the range where it remains constant
- The radius where pixel noise significantly affects the noise pixel calculation
- The radius that contains all significant signal from the source.

For the focus data collected in the post-vibe blue tube test, the noise pixels were measured in a circle up to 12 pixels out from the source. Radial plots show, and the near angle scatter data confirms, that this contains all significant signal from the source without adding significant pixel noise. (In general, it is possible for noise to become significant at a radius too small to contain all significant signal. If so, noise pixels should be calculated over a smaller radius and a correction applied based on other data such as near angle scatter measurements. This is not required here.) The background, which is subtracted from source pixels, is measured well in an annulus 20 to 30 pixels out from the source.

The approximate minimum noise pixels for each focus curve collected in the post-vibe blue tube test and range in values is shown in the table below. The plot below is focus curve 13. At each collimator aperture position (x-axis), five images were collected and noise pixels calculated for each image. The plot shows clear variability in the noise pixel measurement.



Curve	FP Location	Min noise pix	Range p-p	Min with non-linearity
1	Center	19	2.7	
2	Down Left	15.5	1.9	
3	Down Left	14.5	2	
4	Down Right	14.5	1.5	

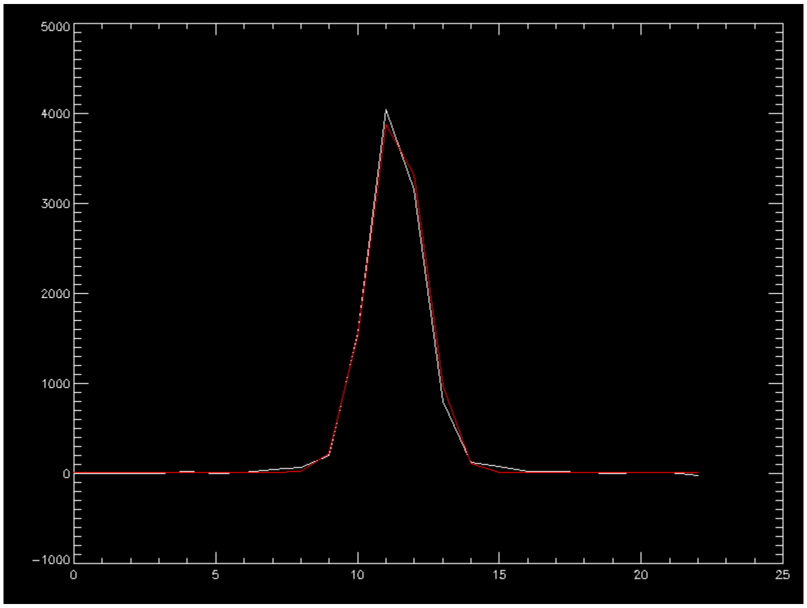
Curve	FP Location	Min noise pix	Range p-p	Min with non-linearity
5	Down Right	16.5	3.6	
6	Center	16.7	3.4	
7	Center	17.5	4	
8	Up Right	17	4	
9	Up Right	18	3.3	
10	Up Left	18.5	5	
11	Up Left	18	1.2	
12	Center	15.7	2.2	
13	Center	17.4	2.1	16.4
14	Center	17.6	1.5	
15	Center	16.5	1.6	
16	Center	16	1	
17	Down Left	16	2	
18	Down Right	14.7	2	
19	Up Right	17.5	2.5	
20	Up Left	19.5	2	
21	Center	18.7	3.7	17.6

The focal plane location is source location as seen if your eye were at the wise entrance and looking out at the source. The min noise pixel value is an approximate average at the position where noise pixels are minimized. The min with non-linearity is the minimum noise pixel value if the non-linearity correction is applied (this was not generally applied to the focus data but was used for the curves shown here). As cases 13 and 21 above show, the non-linearity correction reduces the noise pixels by about 1.

The variability in the noise pixels seen from image to image is clearly due to motion of the spot. From image to image, the spot can be seen to clearly change shape and move slightly. (Radial plots and pixel noise levels show that other sources of uncertainty in the noise pixel measurement are not significant at the level observed here.) The fact that the minimum varies widely for curves at the same location shows the amount of spot jitter is not constant over time. This was clearly observed during LUPI measurements of the collimator focus. On average, the spot jitter was notably worse during this Blue Tube test than during prior tests.

Spot jitter limits accurate measurement of the WISE image quality. The minimum values from the focus curves are 13.5 to 14 noise pixels; applying the linearity correction would reduce this to a bit below 13. It is reasonable to believe this is an upper limit on WISE noise pixels.

It is worth noting here that the spot shapes at best focus are generally circular (in some cases elliptical because of more jitter on one axis) and are always well fit by a Gaussian. For this to be the case, the jitter must be much faster than the 1.1 second integration time used here.



This figure is a line cut in x through the middle of the spot in the first image with the collimator at 0.900” in focus curve 13. The red line is a Gaussian fit.

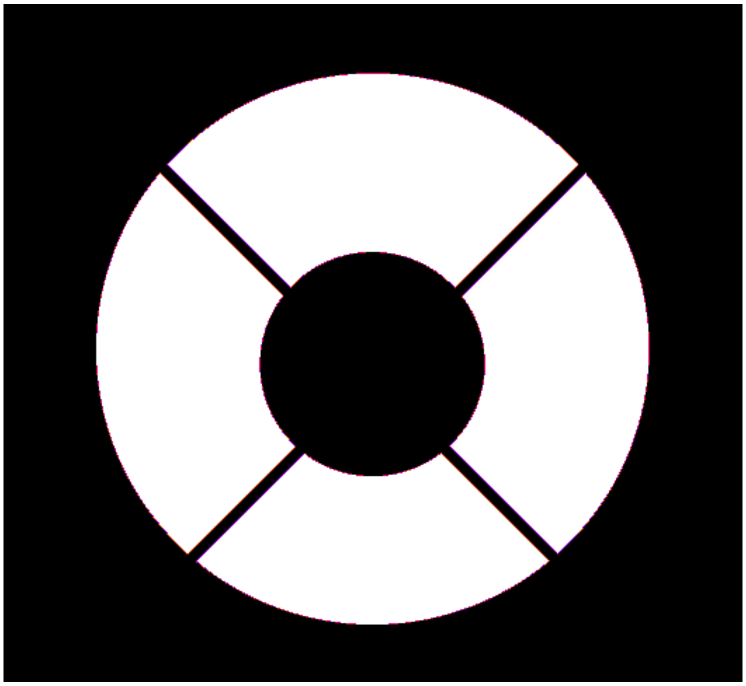
The source of the jitter is unclear. Possibilities are vibrations from the room, vibration due to helium flow through WISE or air currents in the room distorting the beam.

2. FOCUS CURVE MODELING

The expected point spread function can be modeled from the aperture and the expected wavefront of the optical system. The point spread function is defined as

$$PSF(\theta,\phi) = \left| FT\left[A(x,y)e^{i2\pi W(x,y)}\right]\right|^2$$

FT is the Fourier transform, *A(x,y)* is an aperture map (1 where the aperture is open, 0 elsewhere) and *W(x,y)* is the wavefront map. The WISE aperture map is shown below.



This equation calculates the point spread function at one wavelength. The pixel size in the output PSF is given by

$$\theta_{step} = \frac{\lambda}{x_{step} N_{xsteps}}$$

The wavelength chosen is λ, x_{step} is the pixel scale for the aperture and wavefront maps, and N is the number of pixels in the aperture and wavefront maps. The aperture map shown above is on a scale of 0.04" inches per pixel. Calculating results for the band 1 center wavelength of 3.3 µm and choosing N=1218 will give a pixel spacing in the calculated PRF of 0.55 arc seconds, which is 0.2 WISE pixels.

Given the known aperture map and an observed focus curve, the wavefront required to produce the observed focus curve spots can be estimated. There may seem to be too many variables to rigorously do this, but there are simplifying factors. First of all the Fourier transform of the aperture times the wavefront in the complex exponential is the convolution of the Fourier transform of the aperture (this is approximately an Airy disk) convolved with the Fourier transform of the complex exponential.

$$PSF(\theta,\phi) = \left| FT\left[A(x,y)\right] * FT\left[e^{i2\pi W(x,y)}\right] \right|^2$$

Wavefront errors broaden the spot. A general property of Fourier transforms is that small scale structure in a function transforms into large scale structure. Wavefront error with small scale structure (i.e. high spatial frequency variation) transform into a broad function, and thus contributes to power scattered far from the source, where the smaller the structure scale the farther out the power is scattered. To widen the spot near center requires broad, low order structure in the wavefront, such as the classic low order optical aberrations: astigmatism, coma, and spherical aberration.

To model the spot near center, the low order aberrations alone provide a very good approximation. Since the effect of each aberration is different on the in-focus and out-focus spots it is possible to distinguish between them.

The classic optical aberrations add to the wavefront as shown in the table below. The *r* and *θ* are polar coordinates centered on the aperture. The r_{max} is the aperture radius.

Aberration	Function
Focus	$C(r/r_{max})^2$
Astigmatisms	$C(r/r_{max})^2\cos(2\theta-2\theta_{ast})$
Coma	$C(r/r_{max})^3\cos(\theta-\theta_{coma})$
Spherical Aberration	$C(r/r_{max})^4$

The astigmatism and coma have a directional angle, and these formulas produce two waves peak-to-valley (p-v) across the aperture for C=1. Because of their symmetry, when these aberrations are added in, they cannot be compensated for by changing the focus term. The spherical aberration formula gives one wave p-v for C=1, but here adjusting focus will partially compensate and this adjustment must be included.

One wave p-v of focus (C=1 above) corresponds to a distance change at the focal plane of

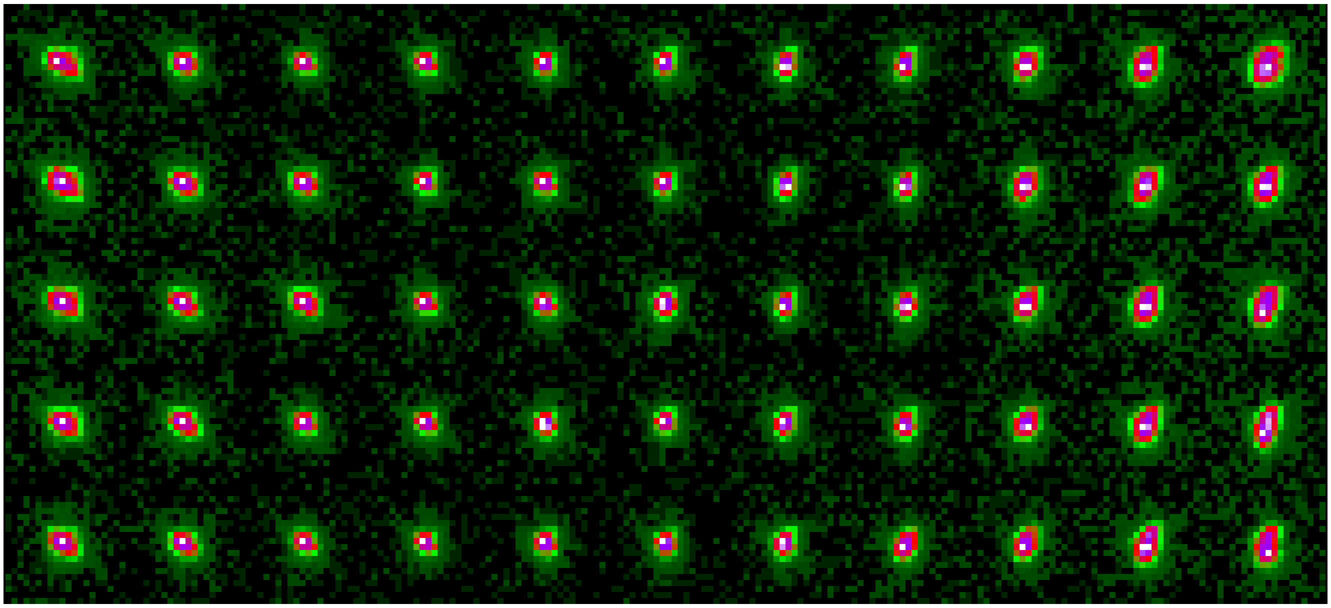
$$d = 8\lambda f_{\#}^2$$

For WISE, this is 300.7 µm. Since the ratio of focus shift at the collimator to that at the WISE focal plane is 33.064, one wave of defocus is 391.4 mils at the collimator.

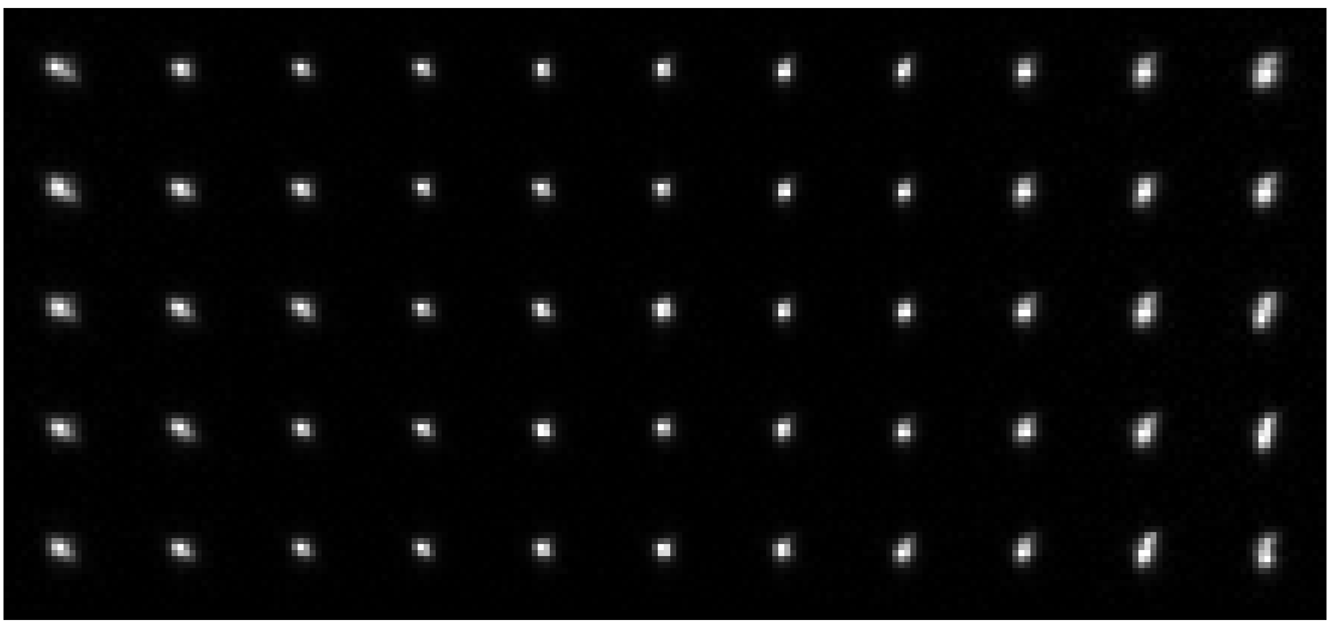
To calculate the predicted PSF, this model is used with various optical aberrations. The following steps are then taken.

1. Include a modeled effect of the Mylar filter used here based on the wavefront variations seen through a Zygo image of clear mylar. This adds a little near angle scatter and is a very small effect
2. Convolve with a 0.49 pixel circle (to model the 50 µm collimator aperture)
3. Convolve with pixel cross talk (1.2 up-down, 1.9% left-right)
4. Rotate to match B1 orientation
5. Shift PSF by an integer number of pixels so that in the re-binned image, the spot center is close to the same fractional pixel position as the data being modeled (to the level allowed by the 1/5 WISE pixel PSF scale output)
6. Combine 5×5 pixels into one to match WISE pixels.

The figure below shows the focus data from focus curve 13.

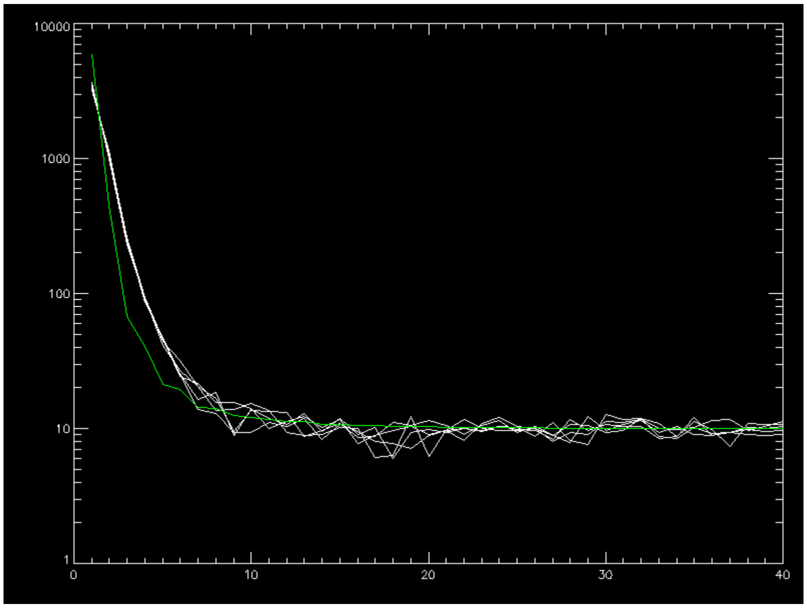


The x axis is collimator position (0.600, 0.660, 0.720, 0.840, 0.900, 0.960, 1.020, 1.080, 1.040, and 1.200) and the y-axis shows the 5 images taken at each position. All spots are normalized by their peak value. From the curve fit to this data (shown above), the best focus is at collimator position 0.869. The black and white image below better shows the central regions.



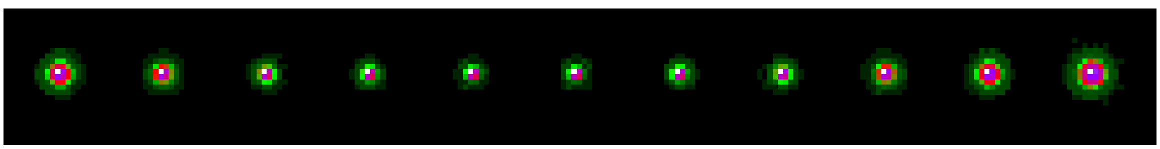
The spot width at best focus varies considerably, but a Gaussian fit to the spot has a 2.2 pix FWHM on average.

The following data shows a radial plot for the observed spots at collimator position 0.840" (closest data to in focus).



The x axis is radius out from center and the y-axis is signal in the average pixel between x and x-1 out from the center. The white lines are the data for the 5 images. The scale is logarithmic with 10 added to all values. The green line in the above plot is the result from an optical model with no aberrations (only the WISE aperture, collimator aperture and crosstalk effects are included). The total signal on the modeled spot is normalized to match the signal on the observed spots.

The plot below shows the calculated spot at all collimator settings. The focus is changed by 0.153 waves for each step, which corresponds to the 60 mil collimator position step, with best focus (no focus wavefront error) halfway between the 5th and 6th spots.



The number of noise pixels at best focus here is 6.6, but this varies from 3.7 to 6.6 depending on the fractional pixel position of the spot center. FWHM of Gaussian fit to this best focus spot is 1.5 pixels. This again varies with fractional pixel position to as low as 1.1 pixels.

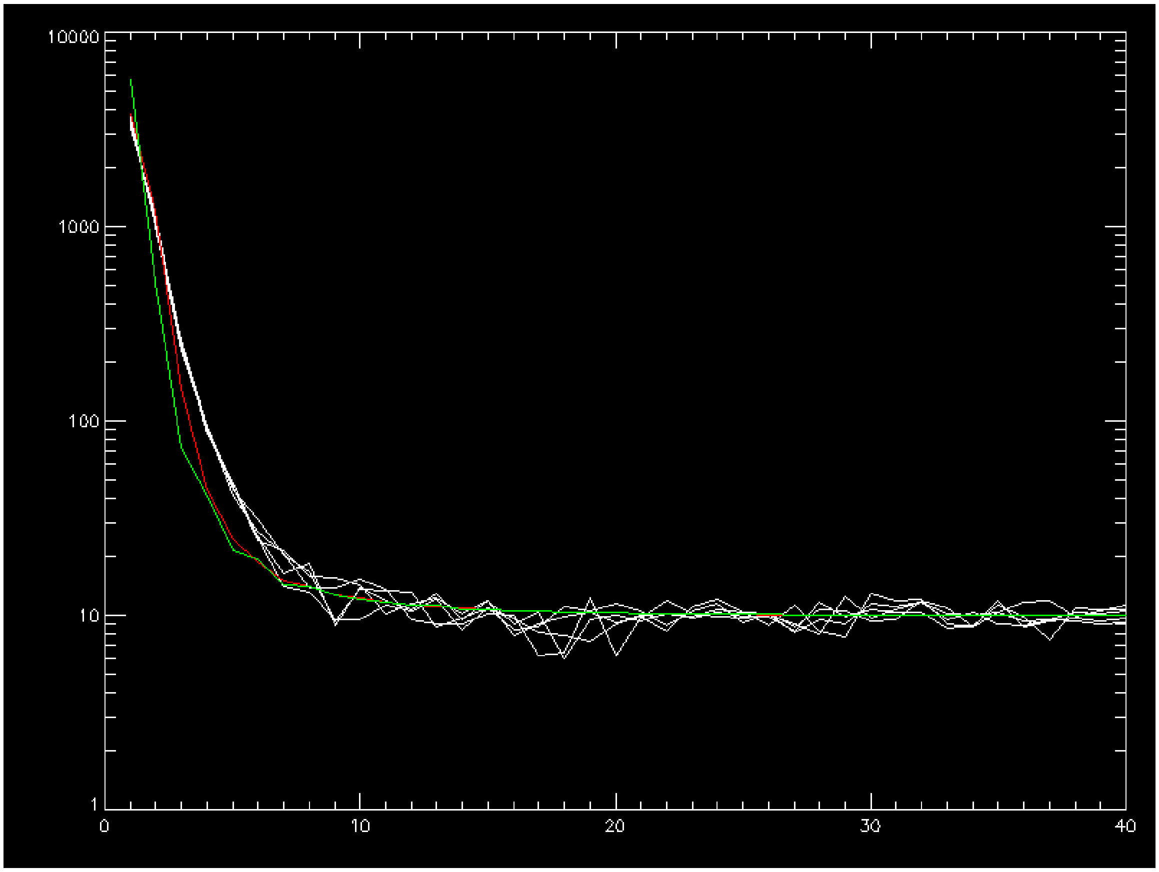
A clear result from this plot is that past a radius of 9 pixels the observed signal on a spot is not much higher than that expected from diffraction. The near-angle scatter data from the third blue tube test also shows little signal outside a ten pixel radius angle scatter. These show that wavefront errors with strong small scale structures are not present in WISE.

2.1 OPTICAL ABERRATIONS AND THEIR EFFECTS

The following sections explicitly show the effects of the low order optical aberrations on the PSF. These models used here include the effects the WISE aperture, the collimator aperture, pixel cross talk, and only the optical aberration specified.

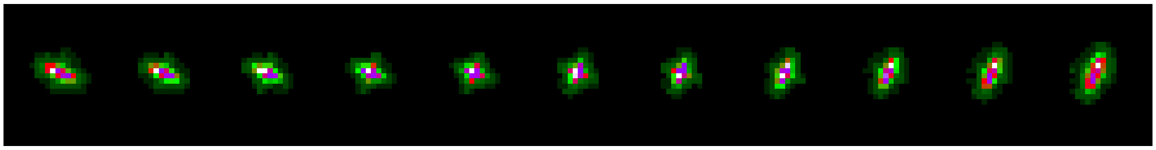
2.1.1 Astigmatism

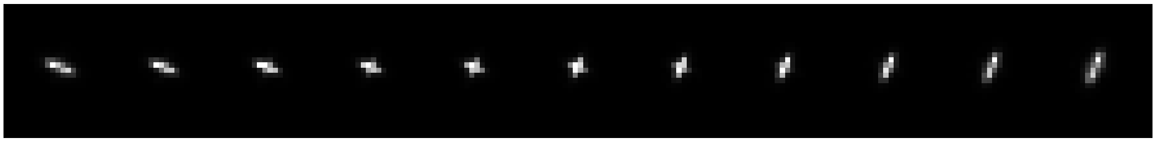
The following shows the radial plot and predicted spot for 1 wave of astigmatism at -22.5° added to the model.



The red line is the new model.

The following are the spots produced from this model.





There are 12.0 noise pixels at best focus, and the width of a Gaussian fit here is 2.2 pixels FWHM.

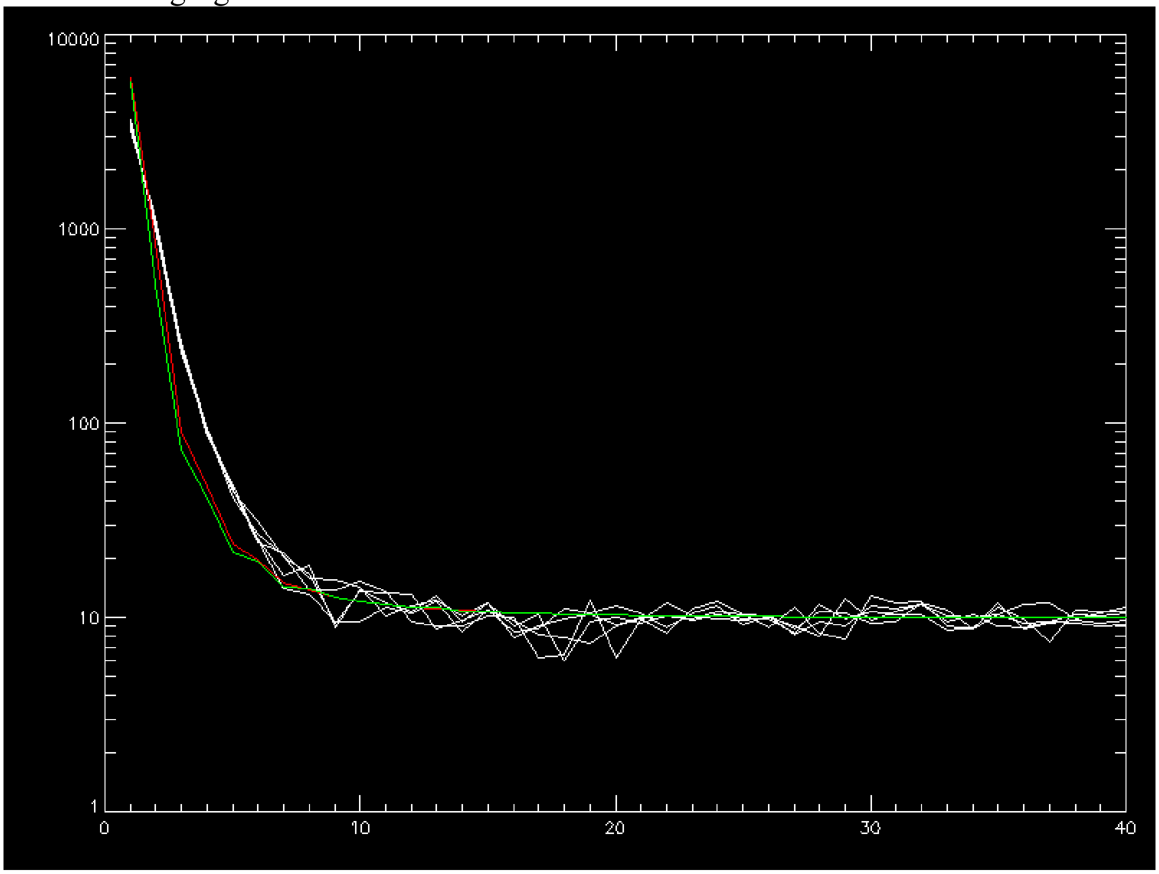
The effects of astigmatism are:

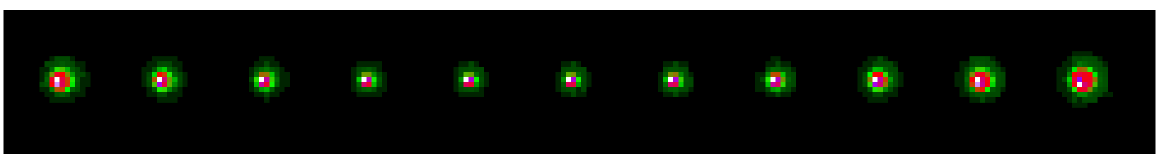
- Elongates out of focus spot
- At best focus

- Adds significant noise pixels
- Puts signal 1-3 pixels out from center of spot, not much elsewhere
- Spot becomes 4-pointed at high enough levels,

2.1.2 Coma

The following figures are for a model with 1.0 wave of coma at 124°.





There are 7.1 noise pixels at best focus, and the width is 1.5 pixels.

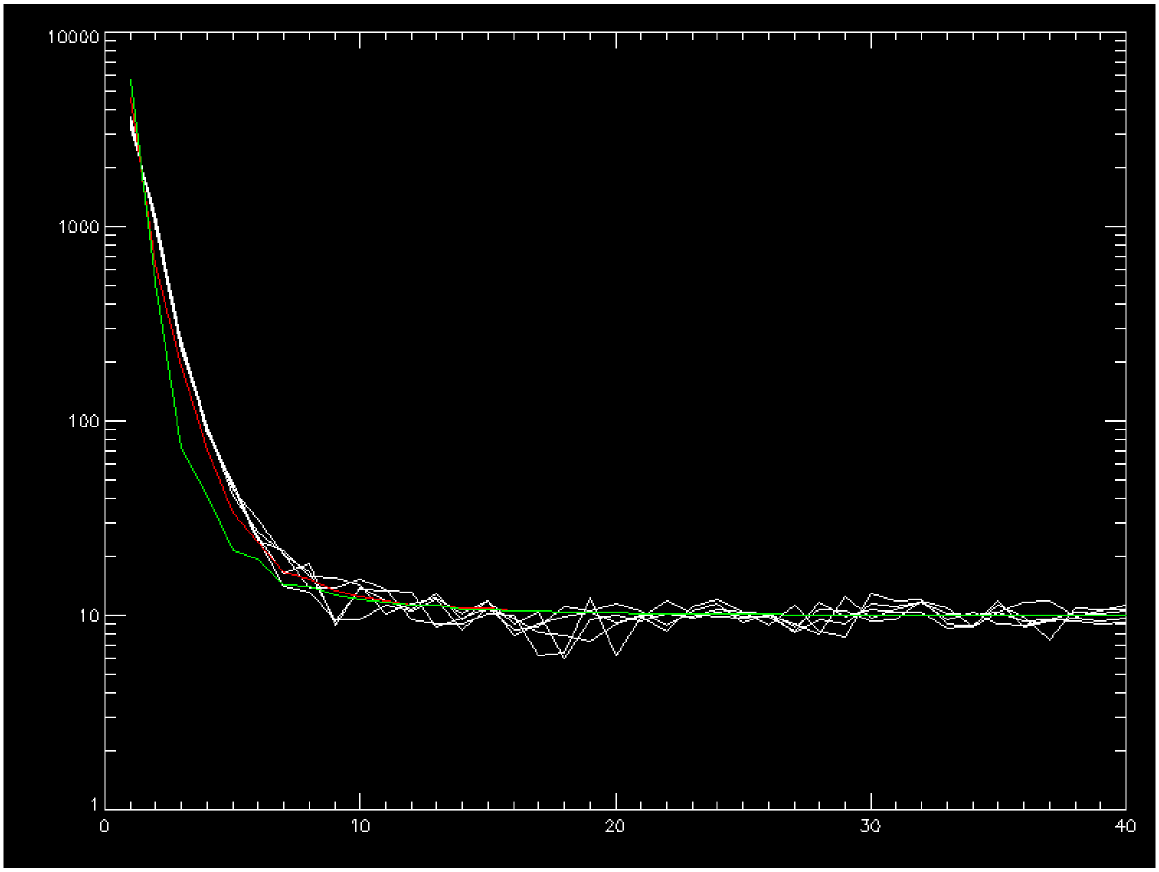
The effects of Coma are:

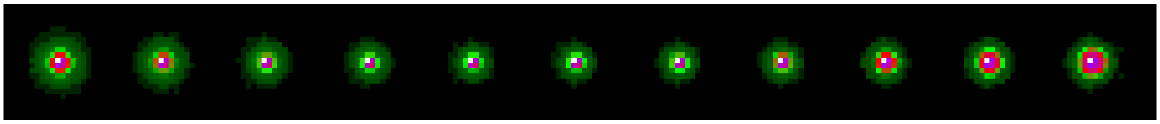
- Makes out of focus spot lopsided
- At best focus
  - Adds little to noise pixels, changes spot little
  - puts a little signal 2-3 pixels out from center of spot

For WISE, the effects of coma are fairly weak. The main change is to make the spot lopsided out of focus.

2.1.3 Spherical Aberration

The following figures are for the model with 1.5 waves of spherical aberration. 1.77 waves of focus are subtracted to compensate.





There are 10.0 noise pixels at best focus and the width is 1.61 pixels FWHM.

The effects of spherical aberration are:

- Slight increase in the wings of out of focus spot.
- At best focus
  - Adds to noise pixels, adds to wings of spot
  - puts signal 2-7 pixels out from center of spot, and a little bit even further out

2.1.4 Higher Order Aberrations

Common sense suggests that higher order aberrations will add power even farther out from the spot than the low order aberrations shown here.

2.2 OPTICAL ABERRATIONS AND WISE

Of the low order aberrations, only astigmatism strongly affects the central regions of the spot and thus the spot widths. Coma has a fairly weak effect on the spot and spherical aberration mostly affects the wings.

WISE cannot have significant higher order aberrations since these would put signal even father from the spot than spherical aberration, and this is not observed.

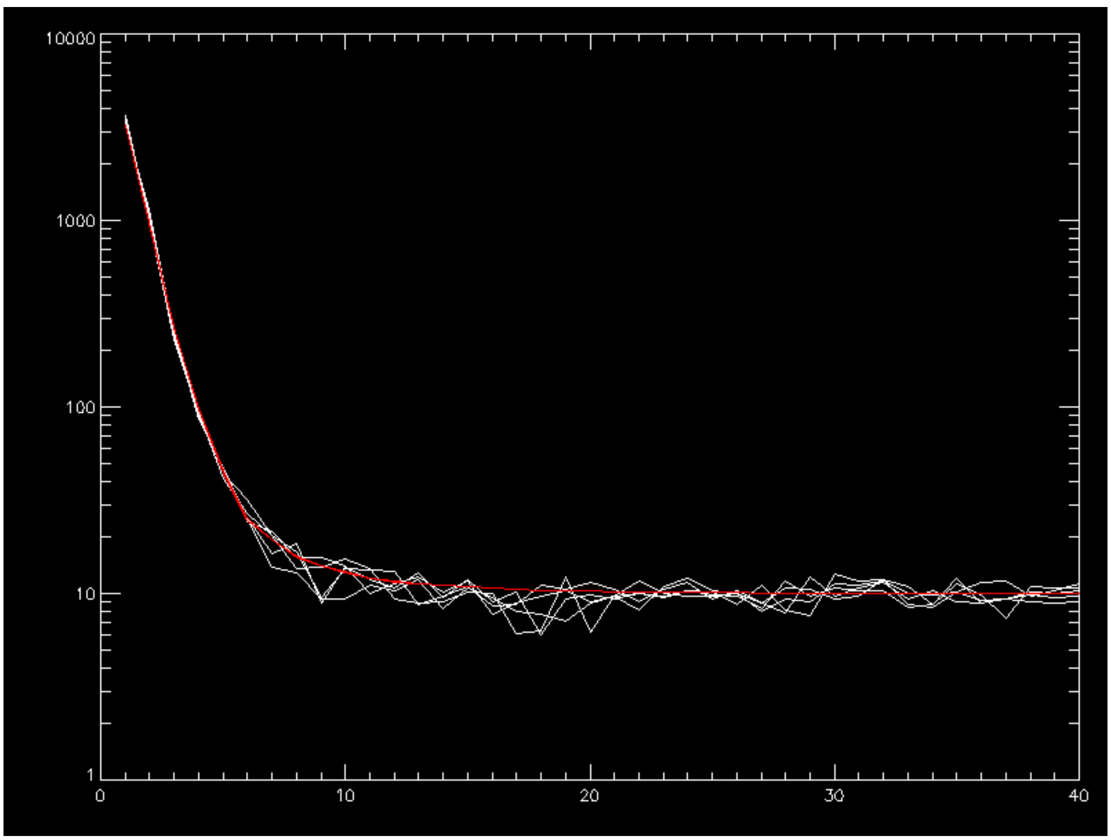
3. RESULTS OF MODELING WISE DATA

In trying to model the WISE data it quickly becomes clear that optical aberrations are not enough to reproduce the observed spots.

The following plots are for a model to match curve 13 with the aberrations

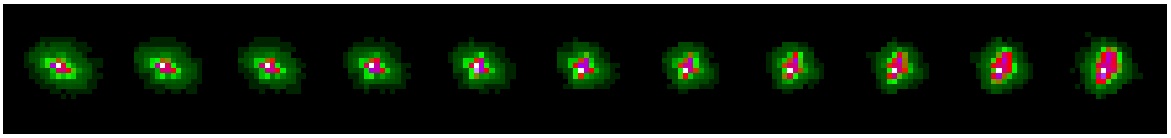
- 0.8 waves astigmatism at -22.5°
- 1.8 waves spherical (-2.18 waves focus)
- 0.8 waves coma at 124°

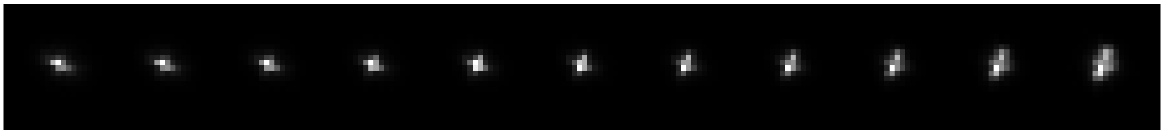
The model and data match well on the radial plot because the values were chosen to make this match well.



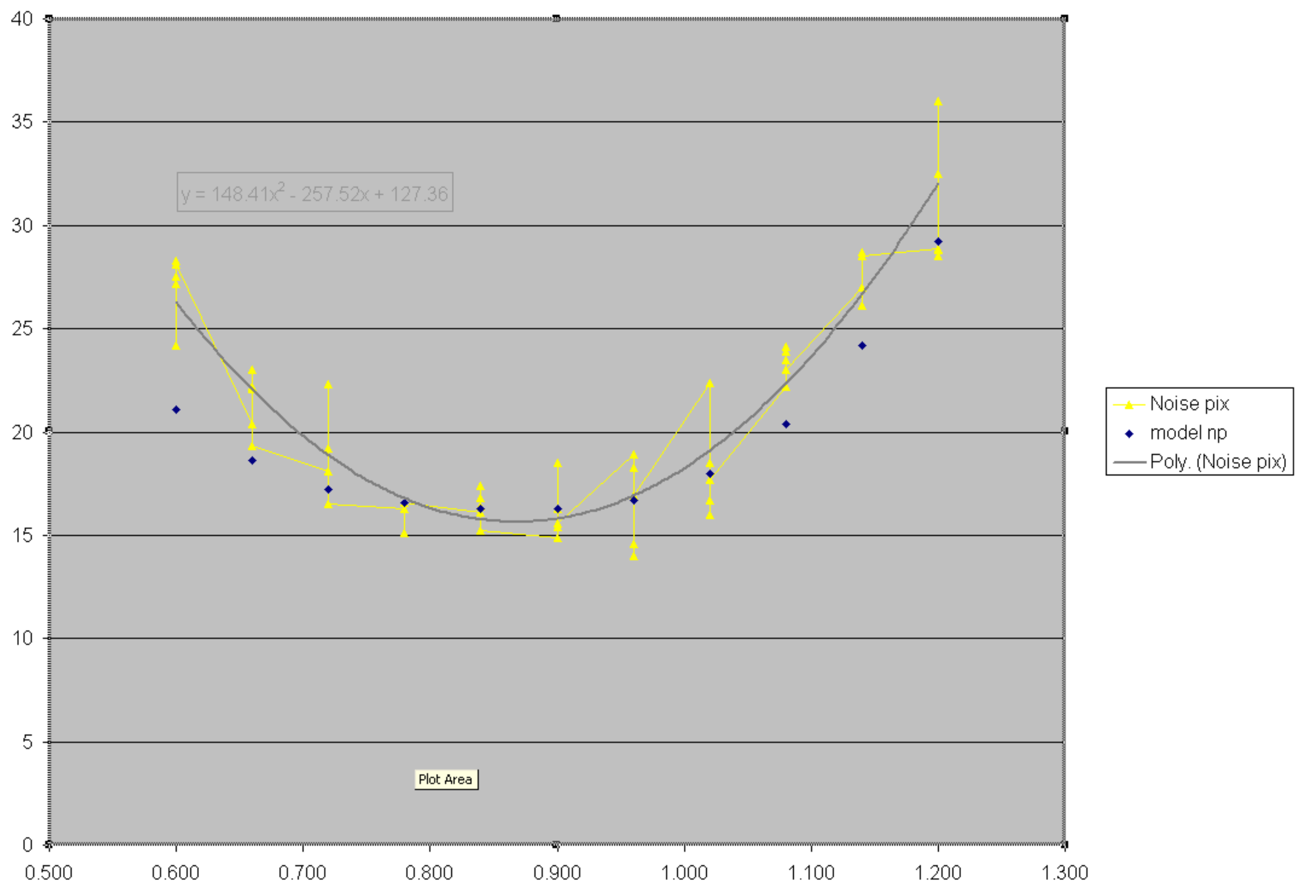
There are 17.1 noise pixels at best focus and the spot FWHM is 2.05 pixels in x, 2.38 in y.

The spot diagrams however are not a good match to the data





The central spot are lopsided, show 4-point structure, and the out of focus spots are too elongated. The noise pixels over the focus curve also do not match the data very well.



A large amount of astigmatism is needed to match the spot width, but this much astigmatism makes the out-of-focus spot too elongated and the in-focus spot too distorted.

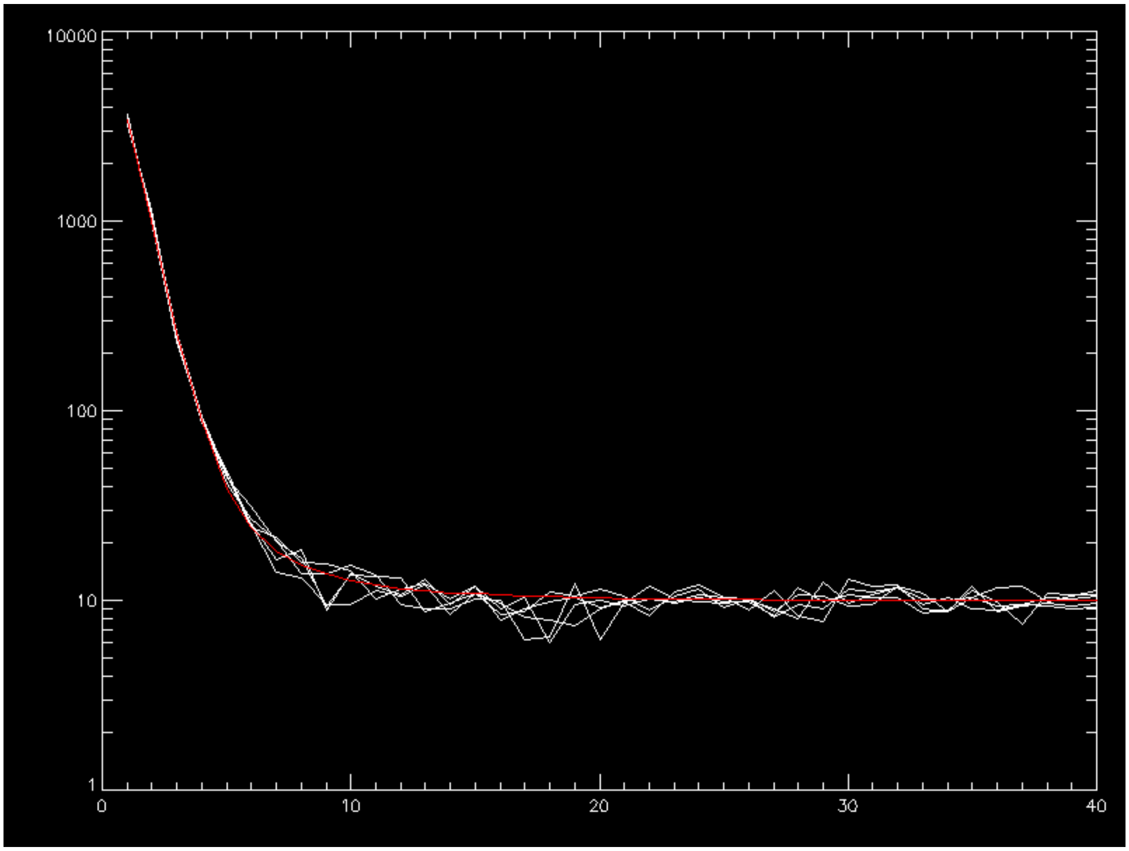
The fact that aberrations alone can not reproduce the observed data set is not surprising since spot jitter has been shown to be a significant effect. To account for this, the result of the optical model was convolved with an adjustable-width Gaussian.

The best model found is

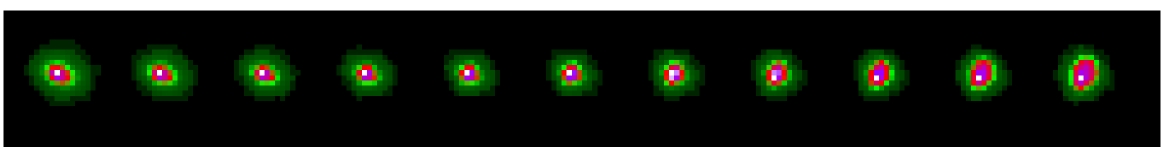
- 0.5 waves astigmatism at -22.5°

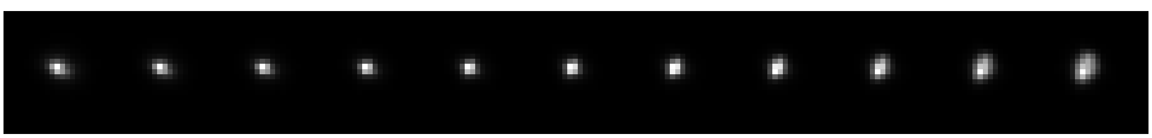
- 1.5 waves spherical (-1.77 waves focus)
- 0.8 waves coma at 124°
- Convolve output with 1.10 pixel FWHM Gaussian

The following plots show the results.

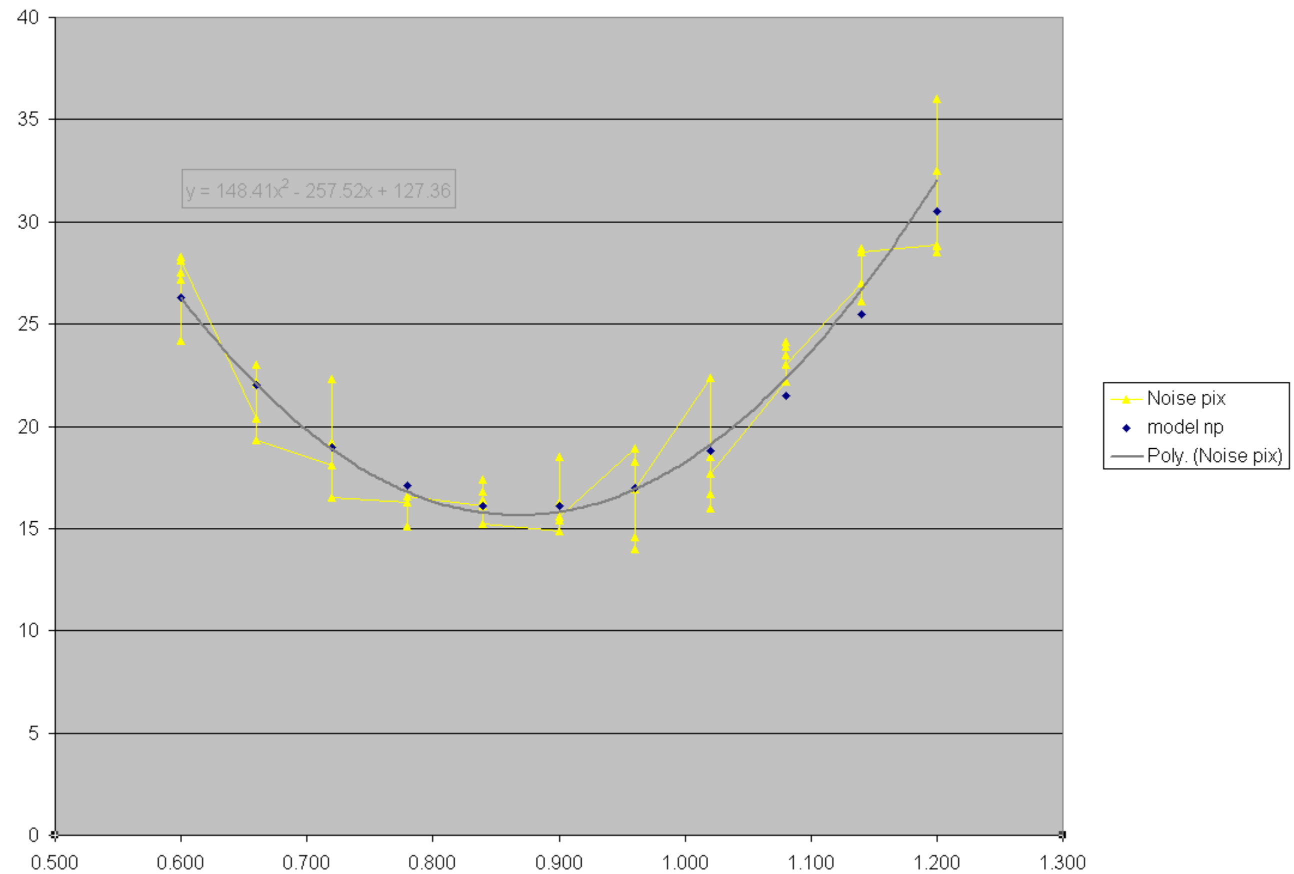


The spot width is 2.2 pix at best focus, which matches the data.





The images look about right as do the noise pixels over the focus curve.



There are 16.2 noise pixels at best focus.

The astigmatism values in conjunction with the Gaussian blur are chosen to match the elongated out of focus spot in shape and in focus spot in width. The direction of astigmatism is determined by the direction of the elongation. The coma is used to match the lopsidedness of the out of focus spot. The spherical aberration along with the Gaussian blur is used to match the radial plot and noise pixel numbers at best focus. The astigmatism value cannot be significantly larger or smaller or the in focus spot shape and out of focus spot elongation will no longer match (estimate this is good to 0.1 wave). The coma can vary by more because it does not strongly effect shape or noise pixels. The spherical aberration affects the number of noise pixels, so its range is limited (estimated at 0.1 to 0.2 waves).

This model is a good match to within the limits of this method. Improving the fit is clearly limited by the variability in the observed spot and the known change in jitter over time. In addition, a small amount of higher order optical aberration may be present that this model excludes. It is not clear that convolution with a Gaussian is correct, though given the match, it cannot be too far off. Further, the model output is for a single wavelength while Band 1 is from 2.8 to 3.8 μm.

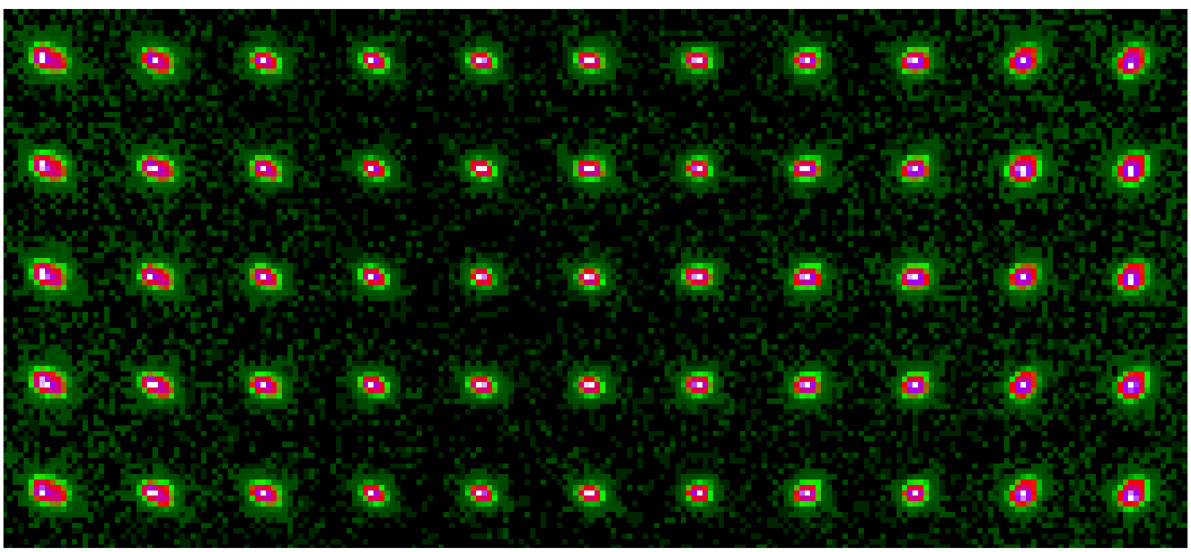
The Gaussian blurring was chosen to match the best average spot size and noise pixels and not the minimum values. If the Gaussian blur is removed, this model predicts 11.9 noise pixels with the spot centered as the data in curve 13, but as low as 10.7 if the model spot is well-centered on a pixel. To be clear, this is the result with just the Gaussian blurring excluded, the collimator aperture and WISE cross talk are still included, and any aberrations from the collimator are not separated out. Since the minimum in the data here is ~15 noise pixels, clearly some Gaussian blurring must always be included to match the data in this curve.

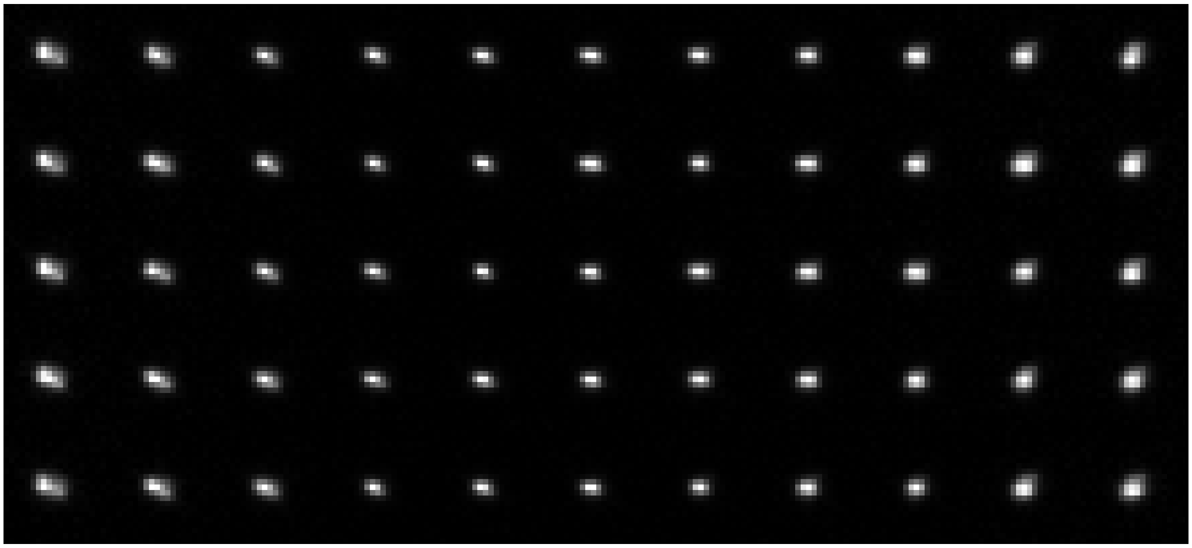
The obvious question is whether the Gaussian blurring can be entirely caused by spot jitter from the WISE ground operational environment. That some blurring is always present is reasonable given the observed variation in spot jitter, but such a blurring must be fast compared to the 1.1 second WISE integration time and generally comparable on each axis. Besides the ground environment, however, it is not clear what can cause this blurring. Based on the optical modeling, optical aberrations can not produce a Gaussian blur. Reduction of the aperture will look like a Gaussian blur since an Airy disk is similar in shape to a Gaussian. A small amount of vignetting may be present for some field positions in the Blue tube test, but testing for vignetting showed there was little, if any. The aperture would have to be reduced by about half for it to be a significant source of blurring.

Assuming this model is correct and all the Gaussian blurring is caused by the ground operational environment, the predicted optical performance of WISE with the collimator used here without this blur is 10.7 noise pixels.

The optical aberration result here is comparable to that from the pre-vibe Blue Tube data.

During collection of focus curve 21 in the post-vibe test, the spot jitter is clearly different with the spots stretched out more on one axis. The spot width is 3.0 (x) and 1.9 (y) FWHM.

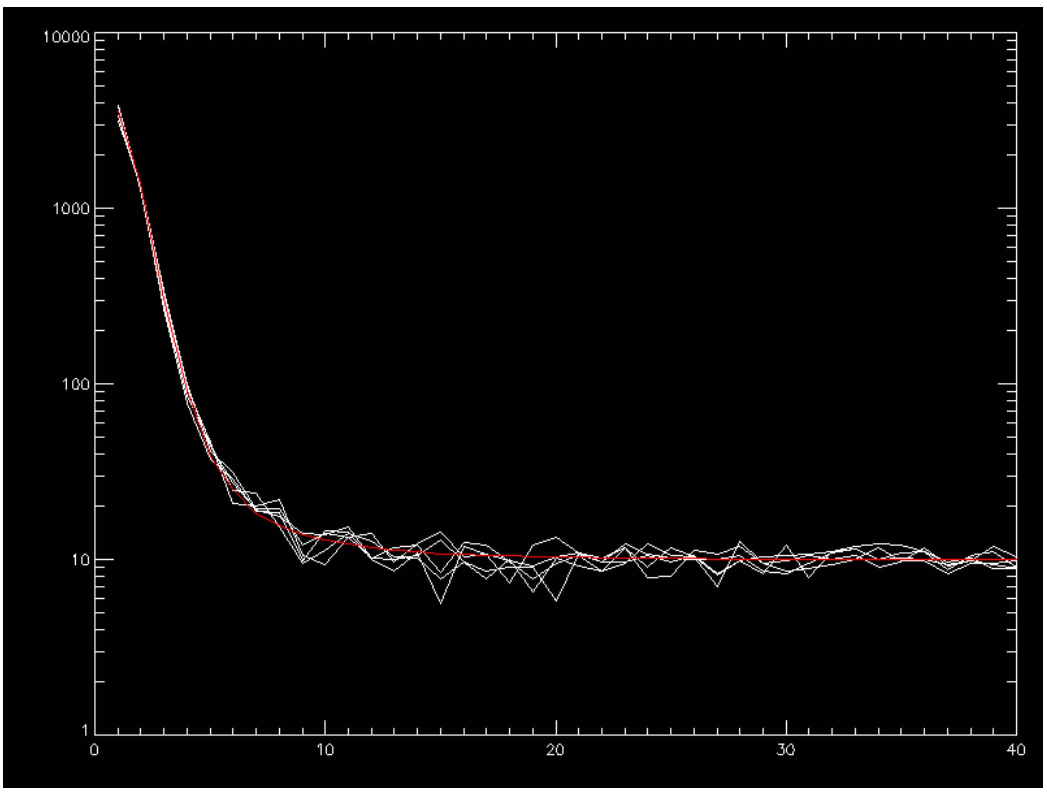


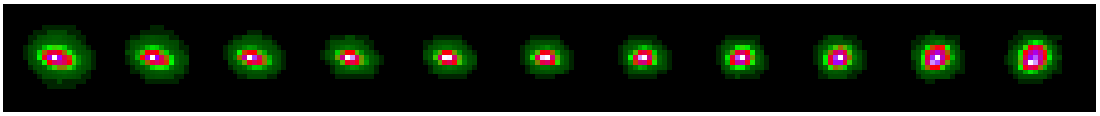


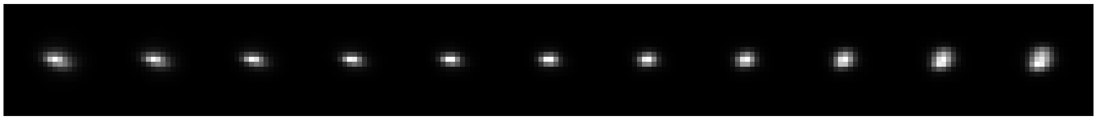
Curve 21 was modeled using:

- 0.5 waves astigmatism at -22.5°
- 0.8 waves coma at 124°
- 1.1 waves spherical with minus 1.25 waves focus to compensate
- Convolution with 2.33 (x) by 0.67 (y) pixel FWHM Gaussian

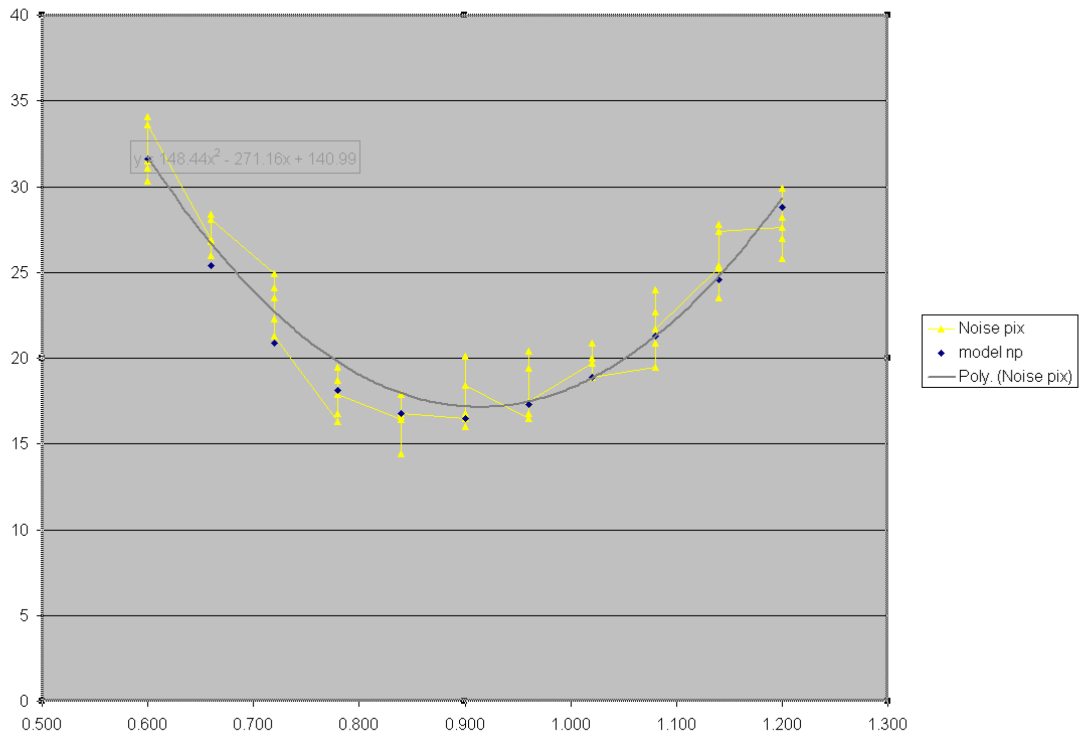
The following plots show the results:







Spot FWHM is 3.0 (x) and 1.8 (y) which matches data. The noise pixels match fairly well, too.



The uncertainty in fitting this curve is greater than the other because of the much larger spot jitter. However, the optical aberrations required here are similar to those for curve 13.

4. CONCLUSIONS

The WISE payload was tested using the blue tube configuration subsequent to vibration testing. The PRF's observed in the post-vibe blue tube test were similar to those observed pre-vibe. The jitter in the blue tube configuration was higher in the post-vibe test than it was during the pre-vibe test. Focus remained unchanged, and low order aberration terms were nearly identical to that obtained in pre-vibe data.

Assuming that ground testing effects such as jitter are the cause of the additional Gaussian component of the measured PRFs, the image quality is estimated to be approximately 10.7 noise pixels including the effects of the blue tube set-up.