SPATIAL DISTRIBUTION MODELS OF THE GALAXY

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In this document a description of contemporary models simulating the spatial distribution of stars in the Galaxy is presented, including a version of the "standard" model used in Jarrett (1992). Our model (and those like it) attempt to predict the stellar surface density in a given direction from the Sun. Specifically, the model predicts the total number of stars per square degree, brighter than apparent magnitude m_λ in a direction whose Galactic coordinates are (ℓ,b). This quantity is usually designated $N(m_\lambda,\ell,b)$. The stellar surface density is determined by the spatial contributions from the Galactic disk and spheroid populations. Accordingly,

$$N(m_\lambda,\ell,b) = N_{disk}(m_\lambda,\ell,b) + N_{sph}(m_\lambda,\ell,b) \ .$$

In addition to disk and spheroid stars, some models include components due to a faint halo population and a "thick" disk population.

Numerical distribution models have been developed by Elias (1978) and Bahcall and Soneira (1980; henceforth B&S) assuming a purely exponential density of stars in the disk (Elias applied this model to near-infrared wavelengths) and in the spheroid (B&S added this component to their optical model). Later Jones *et al.* (1981) and Garwood and Jones (1987) modified the Elias (1978) model to include a spheroidal component, so that their infrared simulation resembled the B&S model.

The text is organized as follows. In the first section, a review of the Galactic stellar distribution, including the disk and spheroid populations, is given. A brief discussion of the controversial thick disk model is also included. In Section 2, we discuss the effects of interstellar extinction and its application within the standard model. A detailed discussion of the model and numerical algorithm used in the thesis of Jarrett (1992) is presented in Section 3. In the last section, we validate the stellar distribution model by comparing the results of the model, applied toward several diverse directions, with observed data and the B&S model.

1. The Stellar Distributions

Seen from the vantage point of the Sun, our home Galaxy is clearly a flattened structure, with the Solar system situated within a disk of stars, and well offset from the center of this spiral "wheel". It is more difficult to view stars above or below the Galactic disk (globular clusters are a notable exception) especially toward the Galactic center, so we must look to nearby galaxies in order to infer the distributions of other spatial components that may be important in building a model of our galaxy.

De Vaucouleurs (1958; 1959; 1977) reported a spheroidal population of stars in spiral galaxies; these stars have a distribution not unlike that of elliptical galaxies that is well-approximated by a flattened exponential. This component is further divided into an inner "bulge", which can also be seen in the Milky Way at far-infrared wavelengths (*cf.* Habing *et al.* 1985), extending ~3 kpc from the Galactic center. A third component, the so-called "halo", has been proposed as a solution to the non-Keplerian, flat rotation curves observed in our Galaxy and in other spiral galaxies (*cf.* Bahcall 1984 for a discussion on the mass to light deficit). However, while halo stars are critical to dynamical models, their intrinsic faintness moderates their overall importance as a spatial component; thus, they are usually omitted from distribution models. We are left with a two component model of the Galaxy, containing an exponential disk and spheroid.

1.1 The Galactic Disk Contribution

The disk population is composed of a mixed ensemble of main sequence stars and evolved stars of varying mass and age. Each spectral type belonging to the dwarf and giant populations[1] can be characterized by a scale height, which is basically an indicator of age. The older the star, the more dynamical interactions it has suffered with other stars and giant molecular clouds (Spitzer and Schwarzschild 1951). The result is an increase in the spatial velocity of older stars (particularly along the vertical axis of the disk). Along with dwarfs and giants, we include in our disk model oxygen-rich Miras, semi-regular and irregular variables, carbon stars, and AGB stars; since luminous *mass-loss* stars have similar photometric and spatial properties (*e.g.*, luminosity, infrared color excess, scale-height), these stars are assembled into one population which we loosly call AGB.

At fixed Galacticentric distance, the number density of stars of spectral type s along the vertical or z direction can be written as

$$\rho_{\perp}(z,s) \propto \exp\left[-\frac{z}{\beta(s)}\right] \qquad \textbf{(1)}$$

where the scale height, β, is a function of spectral type. For example, M dwarfs have relatively large scale heights, β ~ 300 pc, in contrast to the younger A-type stars with β ~ 100 pc (Faber *et al.* 1976). Giants and subgiants have scale-heights typically between 200 and 500 pc, though their uncertainty is greater than that of the dwarf luminosity class (*cf.* B&S).

[1] To simplify the numerical model, distinctions between the differing luminosity classes are not made. Instead, we designate generic dwarf and giant populations. This assumption is adequate, since the total number density of stars in the Galaxy is dominated by luminosity classes V (dwarfs) and III (giants).

Within the plane of the disk, B&S express the radial density function similarly,

$$\rho_1(x) \propto \exp\left[-\frac{(x-R_0)}{\alpha}\right] \tag{2}$$

where x is the distance from the Galactic center (in the plane of the disk), R_0 is the Solar distance from the center, and α is the Galactic disk scale length. The latter parameter varies as a function of galactic morphological type (Freeman 1970). For the Milky Way, published values of α range from 2.2 to 3.5 kpc (De Vaucouleurs and Pence 1978; Knapp *et al.* 1978; McCuskey 1969). B&S adopt a mean value of 3.5 kpc (with R_0 = 8 kpc), whereas Jones *et al.* (1981) treat the scale length as a free parameter ranging from 2.2 to 3 kpc (where R_0 = 8.7 kpc).

Combining the two density functions we arrive at the disk spatial density function

$$\rho(x,z,s) \propto \exp\left[-\frac{(x-R_0)}{\alpha} - \frac{z}{\beta(s)}\right] \tag{3}$$

To transform Equation 3 into a distribution formula per spectral type, one must normalize using the appropriate luminosity functions. The luminosity function per spectral type can be modelled as a gaussian, where each spectral type has an intrinsic uncertainty in its luminosity or absolute magnitude due to calibration assumptions (*i.e.*, discretely grouping stars together) and systematic errors within the calibration (*cf.* Mihalas and Binney 1981):

$$\Phi(M,s) = \frac{\Phi_0(s)}{\sqrt{2\pi\sigma(s)^2}} \exp\left[\frac{|M - M_0(s)|^2}{2\sigma(s)^2}\right] \tag{4}$$

where M_0 is the mean absolute magnitude per spectral type, σ is the intrinsic dispersion and Φ_0 is the luminosity function in the Solar vicinity (*i.e.*, the local number density per spectral type). Now we may write the disk distribution function in complete form:

$$\rho(x,z,M,s) = \frac{\Phi_0(s)}{\sqrt{2\pi\sigma(s)^2}} \exp\left[\frac{|M - M_0(s)|^2}{2\sigma(s)^2}\right] \exp\left[\frac{-(x - R_0)}{\alpha} - \frac{z}{\beta(s)}\right] \tag{5}$$

1.2 The Spheroid Contribution

Following the example of de Vaucouleurs (1959), B&S characterized the spatial distribution of spheroidal stars in the Galaxy with a $r^{1/4}$ exponential measured radially from the Galactic center, as follows:

$$\rho(r) \propto \left(\frac{r}{r_e}\right)^{-\frac{7}{8}} \exp\left[-7.669\left(\frac{r}{r_e}\right)^{\frac{1}{4}}\right] \qquad \textbf{(6)}$$

where r_e is analogous to a scale length, approximately equal to R_0 / 3 (de Vaucouleurs and Buta 1978). A virtue of this density formula is that it assumes spherical symmetry, while other models involve a more complex oblate symmetry (*cf.* King 1966). In a later paper, Bahcall and Soneira (1984) turn to an oblate distribution with an axis ratio of 0.8, to better fit the observational data.

It is by no means an easy task to determine the local luminosity function for spheroidal stars owing to the Sun's relative position with respect to the spheroid, and the rarity of these stars compared to the disk population. Stars in globular clusters are often used to compare with (or add to) those of the field spheroid ensemble[2]. Schmidt (1975) was able to isolate a sample of sub-dwarf stars, presumably belonging to the spheroid population, because of their remarkable space velocities (100 to 250 km s^{-1}). Although the statistics are poor, Schmidt computed the resultant luminosity function and found that the general shape of the luminosity function belonging to the high proper motion stars resembles that of the disk. The overall number density is, however, about 3 orders of magnitude smaller. Combining the data of Schmidt (1975) and Wielen (1974) with the star count data of Bahcall and Soneira (1980), the spheroidal luminosity function is best described using the shape of the disk luminosity function, normalized with a value between 500^{-1} to 800^{-1} (Bahcall, Schmidt and Soneira 1983), where the larger value corresponds to a spheroid with oblate symmetry. Further tuning the spheroid density function in the vicinity of the Galactic center, Bahcall and Soneira (1984) express the density function in complete form:

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[2] Although globular clusters are located within the realm of the spheroid, they are usually treated as separate entities. However, Bahcall and Soneira (1984) conclude that the luminosity functions of globular clusters and spheroid stars are the same within the accuracy of the data (the exception being a depression "feature" between $2 < M_v < 4$ in the globular cluster luminosity function).

$$\rho(r) = 800^{-1} \left(\frac{r}{R_0}\right)^{-\frac{7}{8}} \exp\left[-10.093\left(\frac{r}{R_0}\right)^{\frac{1}{4}} + 10.093\right] \tag{7}$$
$$\times \left[1 - 0.0867\,(r/R_0)^{-\frac{1}{4}}\right], \quad r \geq 0.03R_0$$

1.3 A Thick Disk?

It should be noted there is a fair amount of controversy as to the overall nature of the spheroid population. Gilmore and Reid (1983) claim that many high proper motion stars are in fact a tracer of a *thick* disk component. They point out that if one increases the disk scale heights (β ~ 1500 pc), the resultant distribution matches that of the subdwarf stars. Their stellar surface density simulations include the disk and spheroid populations, as well as a third, "intermediate" population which contains metal-poor subdwarf stars with a z-distribution characteristic of a thick disk (Reid 1983). However, Bahcall and Soneira (1983; 1984) maintain that Gilmore and Reid did not include evolved spheroid stars in their simulation, consequently arriving at a false conclusion. These giant stars are very bright and very far away from the Sun. If one mistakenly identifies them as dwarfs, their intrinsic brightness is vastly changed, the result being that their placement is ~1000 pc off the disk — mimicking a thick disk — as opposed to ~10 kpc where the giants would lie.

Gilmore and Reid question the overall abundance of evolved spheroid giants, suggesting that selection effects[3] may be deceiving their true numbers. Nonetheless, Bahcall and Soneira (1984) are confident the observational evidence for these subgiants is firm (*cf.* Ratnatunga 1982). The fact remains that the B&S model adequately simulates the observed Galactic distribution in several diverse directions (see Section 4), proving the robust nature of the two-component model. The existence of a thick disk shall probably remain in the sphere of debate for some time. The reader is referred to a spirited discussion between the two camps in a talk by Bahcall and Soneira (1983), and to the comprehensive review by Gilmore, Wyse, and Kuijken (1989) in which the authors argue — based on the most recent observational evidence — for the existence of a thick disk.

2. Interstellar Extinction

A final ingredient to consider in modelling surface density is interstellar extinction. Starlight is absorbed by grains of dust and light at shorter wavelengths is preferentially blocked, resulting in a

[3] Given a small viewing cone/beam, the sample of stars within the beam will be biased toward very distant stars (thus, luminous giants) because of the solid-angle effect.

selective reddening of the starlight. The spatial distribution of dust is usually modeled similarly to that of the young disk population, namely, an exponential with a scale height severely restricted to the plane of the disk (~100 pc; Spitzer 1978). Jones *et al.* (1981) expanded this relation to include an absorption scale length in the disk, α_a, which they determine to be ~ 4 kpc. The equation in differential form follows:

$$\delta A_V(x,z,R) \propto \exp\left[-\frac{(x-R_0)}{\alpha_a} - \frac{z}{\beta_a}\right]\delta R \qquad \textbf{(8)}$$

where R is the distance from the Sun. The normalization constant is the local absorption in magnitudes per kpc. The canonical value is typically quoted between 1 and 2 mag kpc^{-1} at visual wavelengths (*cf.* Elias 1978). The absorption function described above does not take into account dense clumps arising from molecular clouds and giant molecular clouds in the disk. This omission may become a problem when modelling the stellar distribution at low galactic latitudes (b < 10°).

3 Model Developed in this Work

In the following discussion, a version of the model described above used in the dissertation of Jarrett (1992) is outlined. Numerical simulations are carried out to validate the model with the results of B&S and real data. The results are described in the section that follows.

Begin by transforming the number density equations into a frame centered about the Sun. Recall that the variable x is measured from the Galactic center in the plane of the disk, and that R is measured from the Sun to the source (or viewing direction); they are related using galactic coordinates, ℓ and b, as follows:

$$x^2(R) = R_0^2 + R^2\cos^2 b - 2RR_0\cos b\cos l \qquad \textbf{(9)}$$

and z, measured perpendicular to the disk, is related to R,

$$z(R) = R\sin b \qquad \textbf{(10)}$$

The variable r, measured radially from the Galactic center to the source is

$$r^2(R) = x^2(R) + z^2(R) \quad \textbf{(11)}$$

Finally a viewing cone of angular radius θ, extending distance R has volume

$$Vol(R,\theta) = \frac{2\pi}{3} (1 - \cos\theta) R^3 \quad \textbf{(12)}$$

B&S define the luminosity, scale height and local density parameters for each spectral type in analytic form as a matter of numerical convenience. In this study, a discrete format using the values in Elias (1978), Jones *et al.* (1981) and Garwood and Jones (1987) is applied. This allows for more flexibility in modifying or adding new spectral types to the sample. For example, the Wielen luminosity function is added to account for the late M-dwarfs ($M_v > 13$ mag); also included is a Hawkins and Bessell (1988) upturn in the luminosity function at the very faint end ($M_v > 16$), which can be compared with the nominal values given by Wielen. The models developed by Elias, Jones *et al.*, and Garwood and Jones are specifically applied to K-band (2.2 μm) data. The transformation involves color corrections (*i.e.*, V - K) and interpolation of the luminosity function. In the case of evolved stars, the V-band luminosity function given in Mamon and Soneira (1982) is used. The scale height and absolute magnitude dispersion values given by Garwood and Jones remain the same. Table 1 lists the spectral type, brightness, colors, local number density, dispersion and scale height. The optical colors are in the Kron-Cousin system (from Bessell 1990), and the infrared colors are from Koornneef (1983). The spheroid population has the same parameters except their number density decreases by a factor of 800, and owing to their intrinsic metal poor composition, their colors tend to be blue with respect to main sequence dwarfs, as follows (from Mamon and Soneira 1982):

$$\delta(B\text{-}V) \approx 0.15,\ \delta(V\text{-}R) \approx 0.15,\ \delta(V\text{-}I) \approx 0.25,\ \delta(R\text{-}I) \approx 0.1,\ \delta(V\text{-}K) \approx 0.4$$

The spatial distribution model embodies a numerical algorithm that follows 4 basic steps: 1) integrate the surface density functions for each spectral type over a differential volume defined by stepping an incremental distance through the viewing cone (that is, a volume bounded by a specified field of view, whose direction is given by the Galactic coordinates), 2) assign the appropriate mean absolute magnitude and its dispersion to the stars given by step 1, 3) compute the total interstellar absorption within the volume by integrating the extinction surface density function, and 4) calculate the apparent brightness, $5\log d - 5 + A_\lambda + M_\lambda$, for each star given by step 1. It is then a simple matter to derive colors and construct cumulative star counts curves.

Consider the disk population. Integrate the density function, Equation 5, out to some distance d:

$$N_T(d,s) = 2\pi(1 - \cos\theta)\ \frac{\Phi_0(s)}{\sqrt{2\pi\sigma(s)^2}}\ \int_{-\infty}^{\infty} dM\ \exp\left[\frac{|M - M_0(s)|^2}{2\sigma(s)^2}\right] \times \int_0^d dR\ R^2\ \exp\left[\frac{-(x(R) - R_0)}{\alpha} - \frac{z(R)}{\beta(s)}\right] \quad \textbf{(13)}$$

where N_T is the total number of stars of spectral type s contained in the cone volume extending distance d. We do not need to waste computer time integrating the luminosity function, Equation 4, over magnitudes ranging from -∞ to ∞. Instead, we solve for absolute magnitude based on the gaussian luminosity distribution, characterized by the mean absolute magnitude and the intrinsic magnitude dispersion, via a Monte Carlo technique. For a gaussian function,

$$\Delta M(s) = \sigma(s)\sqrt{2\ln\left(\frac{1}{RN1}\right)}\ \cos(2\pi\, RN2) \quad \textbf{(14)}$$
$$M_V(s) = M_0(s) - \Delta M(s)$$

where RN1 and RN2 are random numbers between 0 and 1. The constants used in the models are indicated in Table 1 & 2. The spectral-type parameters for the optical and infrared models are given in Tables 1a and 1b. The parameters adopted for AGB stars are determined from Jura and Kleinmann (1989; 1992; and private communication).

Next integrate the density function out to a distance d + δd. The difference in total number is

$$\Delta N(d^*) = N_T(d+\delta d) - N_T(d) \quad \textbf{(15)}$$

We now integrate the interstellar absorption function, Equation 8, out to distance d^*:

$$A_V(d^*) = A_V(0)\ (3/3d^* dR\ \exp\left[-\frac{(x(R) - R_0)}{\alpha_a} - \frac{z(R)}{\beta_a}\right] \quad \textbf{(16)}$$

Table 1a

Spectral-Type Parameters for Optical Model

Spectral Type	M_v	B-V	V-R	R-I	V-K	log Φ_0 kpc^{-3}	σ	β kpc
B1	-3.9	-0.28	-0.11	-0.19	-0.76	2.78	0.5	0.04
B3	-1.9	-0.20	-0.08	-0.13	-0.57	3.52	0.5	0.04
B5	-0.9	-0.10	-0.05	-0.05	-0.43	4.18	0.5	0.04
A0	0.3	0.00	-0.01	-0.02	0.00	4.73	0.5	0.07
A3	1.0	0.30	0.11	0.15	0.20	5.24	0.5	0.07
F5	2.9	0.43	0.27	0.24	1.01	6.10	0.5	0.14
F8	3.5	0.54	0.32	0.28	1.12	6.24	0.5	0.14
G0	4.0	0.60	0.35	0.30	1.22	6.39	0.5	0.22
G3	5.1	0.66	0.37	0.33	1.49	6.52	0.4	0.28
G8	5.6	0.76	0.39	0.37	1.60	6.60	0.4	0.28
K0	6.0	0.80	0.45	0.40	1.75	6.64	0.5	0.30
K3	6.6	0.83	0.47	0.42	2.50	6.72	0.5	0.35
K5	7.6	1.15	0.70	0.57	3.00	6.79	0.5	0.30
M0	8.8	1.41	0.91	0.89	3.25	6.84	0.5	0.30
M1	9.46	1.48	0.94	1.02	3.50	6.87	0.5	0.30
M2	10.14	1.52	0.98	1.16	3.75	6.89	0.5	0.30
M3	11.17	1.55	1.10	1.37	4.00	6.93	0.5	0.35
M4	12.76	1.60	1.23	1.63	4.25	7.17	0.5	0.35
M5	14.70	1.82	1.5	1.9	4.50	7.25	1.0	0.40
M6	17.30	2.00	2.0	2.3	4.75	6.92	1.0	0.40
M7	18.70	2.20	2.1	2.4	5.00	6.82	1.0	0.40
M8	19	?	?	?	5.25	7.04	1.0	0.40
gG0	3.1	0.86	0.46	0.41	2.05	5.11	1.0	0.50
gG5	2.0	0.92	0.48	0.44	2.12	5.31	1.0	0.30
gG8	1.6	0.95	0.49	0.44	2.16	5.19	1.0	0.25
gK1	0.9	1.10	0.57	0.51	2.48	5.08	0.7	0.20
gK3	0.5	1.29	0.69	0.72	2.92	4.85	0.6	0.25
gK5	0.3	1.54	0.84	0.80	3.67	4.70	0.6	0.25
gM0	-0.5	1.56	0.89	0.88	3.74	4.40	0.6	0.25
gM1	-0.9	1.58	0.91	0.92	3.90	4.22	0.6	0.25
gM2	-1.5	1.60	0.95	1.00	4.16	3.90	0.6	0.25
gM3	-2.1	1.62	0.99	1.10	4.63	3.70	0.6	0.25
gM4	-2.5	1.66	1.05	1.20	5.34	3.40	0.6	0.25

Table 2

Model Parameters

	Parameter	Optical	Infrared
Galactic Center	R_0	8 kpc	8.7 kpc
disk scale length	α	3.5 kpc	2.2 kpc
spheroid scale length	r_e	3 kpc	3 kpc
local absorption	$A_V(0)$	2 mag kpc^{-1}	1.5 mag kpc^{-1}
absorption scale height	β_a	0.1 kpc	0.1 kpc
absorption scale length	α_a	4 kpc	4 kpc

The apparent magnitude is given by the distance modulus. For example,

$$\begin{aligned} m_V(s) &= M_V(s) + 10 + 5\log(d^*) + A_V(d^*) \\ m_R(s) &= M_v(s) - (V-R) + 10 + 5\log(d^*) + A_R(d^*) \\ m_I(s) &= M_v(s) - (V-I) + 10 + 5\log(d^*) + A_I(d^*) \end{aligned} \qquad \textbf{(17)}$$

where d^* is measured in kpc, A_R and A_I are derived using reddening curves and the ratio of total to selective extinction. The apparent (reddened) colors are calculated using the above magnitudes. In addition, we may apply a random error to the colors, which will mimic uncertainties due to signal to noise, poor seeing conditions and calibration to standard colors. For example, suppose the random errors for V, R and I are determined to be σ_v, σ_R and σ_I. The color dispersions are then

$$\sigma^2_{(V-R)} = \sigma^2_V + \sigma^2_R$$
$$\sigma^2_{(R-I)} = \sigma^2_R + \sigma^2_I \qquad \textbf{(18)}$$

and the color errors are computed, as before, using a gaussian Monte-Carlo technique:

$$\Delta(V-R) = \sigma_{(V-R)} \sqrt{2\ln\left(\frac{1}{RN1}\right)} \cos(2\pi\, RN2)$$
$$\Delta(R-I) = \sigma_{(R-I)} \sqrt{2\ln\left(\frac{1}{RN3}\right)} \cos(2\pi\, RN4) \qquad \textbf{(19)}$$

where RN1, RN2, RN3 and RN4 are random numbers between 0 and 1.

The simulation using a spheroid component follows the same procedure. Integrating the density function, Equation 7, out to some distance d:

$$N_T(d) = 800^{-1}\, \frac{2\pi(1-\cos\theta)}{\sqrt{2\pi\sigma^2(s)}} \int_{-\infty}^{\infty} dM \exp\left[-\frac{|M - M_0(s)|^2}{2\sigma^2(s)}\right]$$
$$\times \int_0^d dR\, R^2 \left(\frac{r(R)}{R_0}\right)^{-\frac{7}{8}} \exp\left[-10.093\left(\frac{r(R)}{R_0}\right)^{\frac{1}{4}} + 10.093\right] \qquad \textbf{(20)}$$
$$\times \left[1 - 0.0867\left(\frac{r(R)}{R_0}\right)^{-\frac{1}{4}}\right], \qquad r \geq 0.03R_0$$

and integrate out to distance d + δd. Compute the number difference, apply an extinction and color correction. It should be noted that the volumes are integrated out to some maximum distance depending on the direction the model is attempting to simulate. Bahcall, Schmidt and Soneira (1983) tabulate R_{max} as a function of galactic coordinates (see Table 1 in their paper). This value is constrained by the requirement to clearly separate the two major stellar components, the disk and spheroid; the range is approximately given as

$$4 \leq r(R) \leq 12 \text{ kpc}$$

For the disk, the maximum distance to which the surface density function is integrated depends on the Galactic coordinates and geometry of the disk. Here we are assuming a perfectly flat, circular disk of thickness $2z_H$, and diameter 2X. The Milky Way Galaxy appears to have a total diameter of about 25 kpc (*cf.* Bok 1979) and, based on stellar spatial scale heights, a thickness of about 2 kpc or so. The maximum distance, d_{max}, is given within the geometric relation,

$$0 = X^2 - R_0^2 - [d_{\max}\cos b]^2 + 2R_0 d_{\max}\cos b\cos l$$
$$z_{\max} = d_{\max}\sin b \qquad \textbf{(21)}$$

unless z_{max} is greater than z_H, in which case the maximum distance is given by

$$d_{\max} = \frac{z_H}{\sin b} \qquad \textbf{(22)}$$

4 Validating the Model

In this section, a brief description of numerical tests conducted by Bahcall and Soneira (1984) in five different fields is presented. Simulations are performed using the model described in the last section for the fields: 1) north Galactic pole (NGP); 2) SA 57: $\ell = 65°$, b = 86°; 3) SA 68: b = -46°, $\ell = 111°$; 4) Aquarius: b = -57°, $\ell = 36°$; and 5) SA 51: b = +21°, Galactic anticenter, comparing these results with those of Bahcall and Soneira, and real data. The fields were chosen by Bahcall and Soneira to minimize confusion and interstellar extinction (*i.e.*, by avoiding locations anywhere near the plane of the Galaxy), as well as to test the degree of oblateness in the spheroid. Finally, additional fields are considered and the results are compared with infrared data from the 2MASS[4] Protocam. The fields lie in the directions of the Galactic plane ($\ell = 53°$ and 140°, b = 0°), toward M92 ($\ell =$ 68°, b = 34°), and the NGP.

[4] Two Micron All Sky Survey

Both SA 57 and the NGP offer an excellent test of the spheroid parameters because disk stars are considerably less numerous well off the plane, whereas spheroid stars can be seen several kpc from the Sun. Bahcall and Soneira were able to fit their simulated differential starcounts to the data with better than 20% accuracy. They found that spheroid stars begin to dominate the total number observed at visual magnitudes fainter than ~17. Disk giants diminish rapidly beyond V ~ 12, and disk dwarfs vanish at V ~ 25. In Figures 1a and 1b plots of the differential starcounts using the model described in Section 3 is shown for these two regions. The total contribution is drawn with a solid line, disk dwarfs are shown with a short dashed line, disk giants with a dotted line, and the total contribution due to the spheroid population is drawn with long dashed line. Real data is represented by filled circles. In Figure 1a it is apparent that the predicted starcounts fit the data by Reid and Gilmore (1983) with an accuracy ~10% for visual magnitudes fainter than 10. At the bright end the fit begins to deviate substantially, due in part to Poisson statistics. This effect can be seen in Figure 1b as well, where the predicted counts do not follow a smooth line or the data of Weistrop (1972). Overall the fit in Figure 1b is good to ~20%, where some deviation settles in at V > 16 mag. As in the case with the B&S model fits, it can be seen that the spheroid stars begin to dominate the total numbers beginning at V ~ 17. The dominant spectral component in the spheroid population are distant K giants.

As we move down in galactic latitude, the importance of spheroid stars lessens relative to disk dwarfs. This effect can be seen in Figures 2a (SA 68) and 2b (Aquarius). The main difference between the two plots is that the direction of Aquarius lies substantially closer than SA 68 to the Galactic center; consequently, more spheroid stars are seen (*cf.* the "bulge" sub-component of the spheroid). In the case of SA 68, the model fits the data (taken from Bahcall and Soneira 1984) to an accuracy of 10 - 25%, which is about the accuracy of the photometry itself. The predicted counts for Aquarius have an even better fit, ~10%, with only two data points deviating by about 20% (Tritton and Morton, 1983, taken from Bahcall and Soneira 1984). Likewise, B&S were also able to fit the predicted starcounts to the data with accuracies better than 20%.

Finally, consider the case of SA 51. In Figure 3, a plot of the predicted differential starcounts versus observed data (by King, taken from Bahcall and Soneira 1984) is shown. Notice that at low galactic latitudes the total number density increases, while the contributions due to disk giants and spheroid stars substantially decreases. The fit is accurate to ~20% between $12 < V < 21$, with greater deviation occurring at the bright end, (again, due to statistics) and the faint end, where only ~30% accuracy is achieved. The model fits by B&S do a little better at the faint end, with 20% accuracy.

To summarize: both the B&S and the optical spatial distribution model of this study adequately fit, to the accuracy of the observational data, starcounts in five different Galactic fields. The fact that these fields are located toward diverse galactic latitudes and longitudes attests to the robustness of the models. The models can thus be applied, with a satisfactory degree of confidence, toward fields lying between $30° < \ell < 330°$ and $b > 20°$. Beyond these limits there is uncertainty, since no tests were performed by Bahcall and Soneira.

Simulations using the infrared parameters provide similar (re: encouraging) results as those presented above. The NGP model simulations agree quite well with the Protocam data (which has photometric accuracy of 10 - 20%), as seen in Figure 4. Within the Galactic plane the model results (Figures 5 and 6) appear to match the true stellar distributions as evident from the Protocam data (in the luminosity range in which the data is complete). The success of the model in this spatial regime is perhaps surprising given the extreme source density (two orders of magnitude greater than that of the NGP) and the rather simplistic assumptions of the model. Be that as it may, the model results clearly show that evolved stars, and in particular AGB stars (see Figure 5), dominate the total star counts contributed by the aggregate disk population. It is interesting to note that the field in the direction of $\ell = 140°$ is cloaked by a molecular cloud (NGC) that lies some 2.2 kpc from the Earth. The cloud acts as a semi-opaque screen, with visual extinctions ranging from a few to perhaps 50 mag. We have included in our model an opaque 'wall' located 2.2 kpc from the Sun possessing an average extinction of $A_V = 10$. From Figure 6 it can be seen that the cloud tends to dampen the source counts

of distant, luminous evolved stars, while having little effect on the dwarfs (since they are mostly foreground to the cloud). The total yield in source counts agrees to within 10% of the data observed using the SQIID array.

For higher Galactic latitudes ($b > 10°$), the faint source counts are dominated by F - K dwarfs. Consider the high Galactic latitude field toward the M92 globular cluster ($b \sim 34°$). Figure 7a shows the model results along with the Protocam data which includes sources from the globular cluster. Figure 7b is the same as 7a except that the M92-associated stars have been subtracted from the Protocam data. Again, the simulations adequately match (to within 10-15%) the field star population toward the Hercules constellation (Figure 7b).

It should be noted that the photometric data provide information regarding the *total* cumulative star counts, but do not discriminate between various spectral types and luminosity classes. In most cases it is not possible to test the validity of the relative fraction of dwarf to giant to spheroid stars that is provided by the spatial distribution models.

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Figure 1 Differential starcounts *versus* apparent visual magnitude toward fields (a) north Galactic pole, and (b) SA 57, b = 86°, ℓ = 65°. Model predictions drawn with lines according to: solid denotes the total contribution, short dashed the disk dwarfs, dotted the disk giants, and long dashed denotes the total contribution of the spheroid stars. Observed data from Reid and Gilmore (1983) and Weistrop (1972) is represented by filled circles.

Figure 2 Differential starcounts *versus* apparent visual magnitude toward fields (a) SA 68, b = -46°, ℓ = 111°, and (b) Aquarius, b = -57°, ℓ = 36°. Model predictions drawn with lines according to: solid denotes the total contribution, short dashed the disk dwarfs, dotted the disk giants, and long dashed denotes the total contribution of the spheroid stars. Observed data from Bahcall and Soneira (1984) and Tritton and Morton (1983) is represented by filled circles.

Figure 3 Differential starcounts *versus* apparent visual magnitude toward SA 51, b = 21°, ℓ = 181°. Model predictions drawn with lines according to: solid denotes the total contribution, short dashed the disk dwarfs, dotted the disk giants, and long dashed denotes the total contribution of the spheroid stars. Observed data taken from Bahcall and Soneira (1984) is represented by filled circles.