Accuracy in Relative Responsivity Correction

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Below we estimate limits on the accuracy of flat-field products needed to satisfy photometric sensitivity requirements.

We assume a simple aperture photometry model. An estimate of the point source flux can be written:

$$\hat{S} = N_p f_s (s_p + b_s) - f_a N_p b_a, \qquad (1)$$

where

N_p = effective number of noise pixels ("footprint" of a point source)

f_s = mean responsivity correction factor for pixels in source aperture

f_a = mean responsivity correction factor for pixels in background annulus

s_p = mean source flux per pixel before flatfielding

b_s = mean background flux per pixel in source aperture before flatfielding

b_a = mean background flux per pixel in background annulus before flatfielding

Assuming that *only* the responsivity correction introduces error, we rewrite the above equation in terms of true + error (ε) terms:

$$S_t + \varepsilon_{phot} = N_p \left(f_s + \varepsilon_{fs}\right)\left(s_p + b_s\right) - \left(f_a + \varepsilon_{fa}\right) N_p b_a,$$

where

S_t = the true source flux

ε_{fs}, ε_{fa} = error fluctuations in responsivity corrections

Squaring both sides, taking expectation values, and assuming that the background is approximately constant over the aperture-annulus region: $b_s \sim b_a \sim b$, we have a relation between the noise variances in the source-flux and responsivity correction:

$$\sigma_S^2 = \left(N_p s_p\right)^2 \left[1 + 2\left(\frac{b}{s_p}\right)^2\right] \sigma_f^2 .$$

Given the source signal-to-noise ratio $SNR = N_p s_p / \sigma_S$ and the source-sigma per pixel σ_p, we have an upper bound on the 1-sigma uncertainty in the responsivity:

$$\sigma_f < \left[SNR^2 + 2\left(\frac{b}{\sigma_p}\right)^2\right]^{-1/2} . \qquad (2)$$

This is an upper bound since our calculation ignores all other sources of error in the photometry. Here are some interesting limits:

(i) for negligible backgrounds, the error in the responsivity reduces to the relative error in photometry: $1/SNR$, assuming of course no other errors contribute.

(ii) for high backgrounds, the b/σ_p term dominates and we have $\sigma_f \approx \sigma_p / b$, implying $\sigma_p / s_p \approx \sigma_f (b/ s_p)$. Written this way, one can see that if $b > 100 s_p$ and we have a 1% error in the flat, we will get a >100% error in the photometry!

In terms of the expected WISE 5-sigma sensitivities for 8 frame repeats S_{lim}, the source-sigma per pixel σ_p in Eq. (2) can be estimated from:

$$\sigma_p \approx \left(\frac{1}{5}\right)\sqrt{\frac{8}{N_p}} S_{\text{lim}}$$

The table below summarizes upper bounds for the 1-sigma relative error in the responsivity σ_f along with all inputs assumed. These are the maximum errors needed to satisfy our *expected* 5-sigma point-source sensitivity limits. Since other errors are involved, we have assumed that the relative error due to flat-fielding contributes no more than 50% of the total photometric error budget (taken from the WISE Calibration Plan, May 2008). Therefore, we multiplied the values computed from Eq. (2) by 0.5. The contribution is very uncertain, so it's best to be conservative.

Band	**σ_f (%)**	**S_{lim} (μJy)**	**σ_p (μJy/pix)**	**b (μJy/pix)**	**B (MJy/sr)**	***SNR***
1	8.27	80	12.23	29.32	0.165	5
2	3.15	95	13.03	138.99	0.782	5
3	0.44	760	70.96	5741.29	32.300	5
4	0.29	3700	403.55	49556.42	69.700	5