Scan Rates Computed From FITS Position Parameters

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1. Overview

During the WISE In-Orbit-Checkout period, optimal scan rates were sought via computations that involved sampling point-source images at various commanded axis rates and solving for the rates that minimized the extent of the PSFs (see the Technical Memorandum *WSDC-D-T020*, "PSF Moment Parameters and Uncertainties"). It was desired to have estimates of the scan rates about all three axes that were independent of the gyro information, and so an algorithm was designed to compute these for a series of chronologically consecutive FITS images for which pointing reconstruction had been obtained via the PRex subsystem. This document describes that algorithm.

Each FITS image contains pointing information in the header parameters CRVAL1, CRVAL2, and WCROTA2, whose values are the RA, Dec, and twist angle commonly called PA, denoted herein as α, δ, and γ. These can be used to compute array XYZ axis unit vectors in celestial coordinates, and changes in these vectors from one frame to the next can be divided by the time interval between frame snapshot times to obtain scan rates expressed in angles about these axes. *X* and *Y* are image array coordinates, and *Z* is the boresight of the array's look direction. We will be mainly concerned here with the rates about *X* and *Y*, because these are what affect the PSF shape, since the rate about *Z* must be negligible in order to avoid time variation of the *X* and *Y* rates, but we include it in the algorithm.

These rate snapshots are centered on the midpoints between frame times rather than the frame times themselves, but as long as the rates are essentially constant, this does not introduce any problems, and if any significant variation were to occur, it would still be revealed by this computation.

2. *XYZ* Rates Between Two Consecutive Frames

We consider two consecutive frames with position angles $(\alpha_1, \delta_1, \gamma_1)$ and $(\alpha_2, \delta_2, \gamma_2)$ and times T_1 and T_2, respectively. Strictly speaking, the frames need not be consecutive, but such frames were generally available and allowed the finest resolution of the rates. For each frame, suppressing indexes, the following matrix was computed for transforming components of vectors in a celestial coordinate system to frame *XYZ*, where the celestial coordinate system is defined with its X_c axis pointing to the vernal equinox and its Z_c axis pointing to the north pole. The Euler rotation sequence is α-90º about Z_c, δ-90º about the new *X* axis, and γ+180º about the new *Z* axis:

$$\phi_1 \equiv \alpha - 90°$$

$$\phi_2 \equiv \delta - 90°$$

$$\phi_3 \equiv \gamma + 180°$$

$$M \equiv \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}$$

$$M_{11} = \cos\phi_3 \cos\phi_1 - \sin\phi_3 \cos\phi_2 \sin\phi_1$$

$$M_{21} = -\sin\phi_3 \cos\phi_1 - \cos\phi_3 \cos\phi_2 \sin\phi_1$$

$$M_{31} = \sin\phi_2 \sin\phi_1$$

$$M_{12} = \cos\phi_3 \sin\phi_1 + \sin\phi_3 \cos\phi_2 \cos\phi_1$$

$$M_{22} = -\sin\phi_3 \sin\phi_1 + \cos\phi_3 \cos\phi_2 \cos\phi_1$$

$$M_{32} = -\sin\phi_2 \cos\phi_1$$

$$M_{13} = \sin\phi_3 \sin\phi_2$$

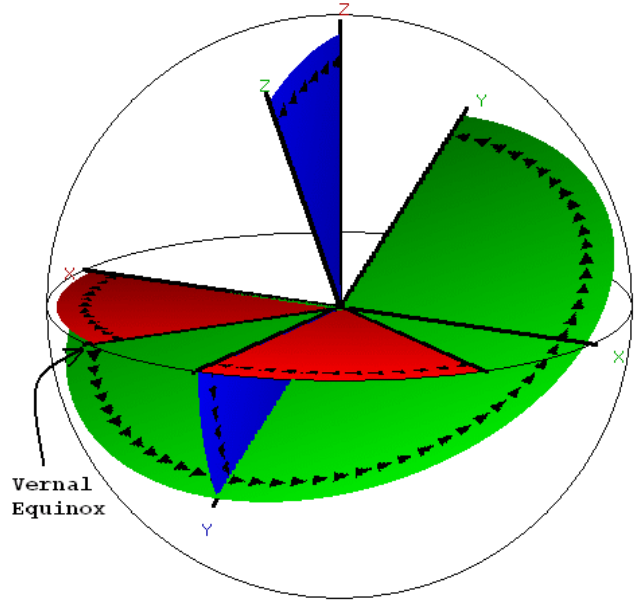$$M_{23} = \cos\phi_3 \sin\phi_2$$

$$M_{33} = \cos\phi_2$$

Illustration: $\alpha = 30°$, $\delta = 60°$, $\gamma = 0°$
1.) α-90º = -60º about Z
2.) δ-90º = -30º about X
3.) γ+180º = 180º about Z

The rows of this matrix are the unit vectors of the frame XYZ system axes in the celestial coordinate system. Since the Z axis is the array's boresight (actually we have yet to account for optical inversion; we will do that below), we are interested in how the Z axis of one frame has changed when represented in a given (different) reference frame. This change has components on the reference frame's X and Y axes, and angle equivalents of those components divided by the time interval are the rates we seek on those axes. Taking T_2 as the later time, we want the components of a unit vector on Z_2 expressed in the $X_1Y_1Z_1$ system. We define the unit vector on the Z_2 axis as follows:

$$\hat{z}_2 = \begin{pmatrix} M_{31} \\ M_{32} \\ M_{33} \end{pmatrix}_2$$

That is, the unit vector on the Z axis of frame 2 has celestial components equal to the third row of the transformation matrix for $(\alpha_2, \delta_2, \gamma_2)$. We rotate this into the frame-1 system:

$$\hat{z}_{21} = M_1 \hat{z}_2 \equiv \begin{pmatrix} X_{z21} \\ Y_{z21} \\ Z_{z21} \end{pmatrix}$$

where the subscript "21" indicates a unit vector on a frame-2 axis mapped into frame-1 coordinates. The angular changes of the frame boresight relative to the frame-1 X and Y axes are:

$$\Delta X = \tan^{-1}\left(\frac{X_{z21}}{Z_{z21}}\right)$$

$$\Delta Y = \tan^{-1}\left(\frac{Y_{z21}}{Z_{z21}}\right)$$

To get the rate *about* the Z axis, we map the frame-2 X axis into frame-1 coordinates (we could also use the Y axis) and examine the angle it makes relative to the X axis in that coordinate system. The unit vector on the frame-2 X axis is the first row of that matrix:

$$\hat{x}_2 = \begin{pmatrix} M_{11} \\ M_{12} \\ M_{13} \end{pmatrix}_2$$

We rotate this into the frame-1 system:

$$\hat{x}_{21} = M_1\,\hat{x}_2 \equiv \begin{pmatrix} X_{x21} \\ Y_{x21} \\ Z_{x21} \end{pmatrix}$$

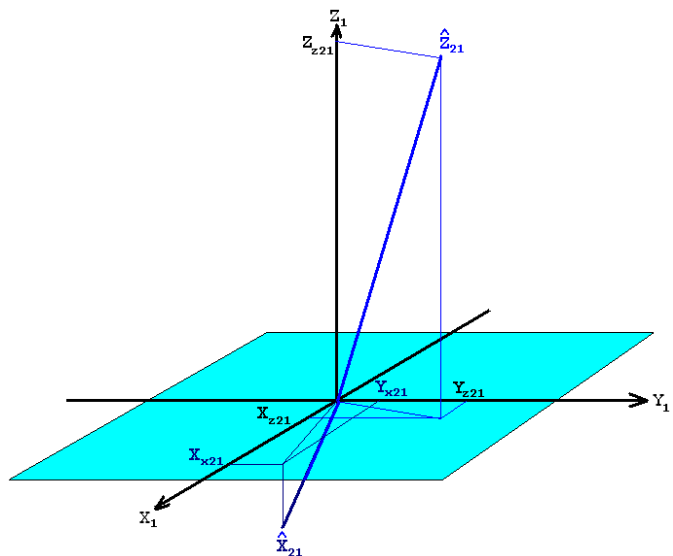The angle this makes with respect to the frame-2 X axis is:

$$\Delta Z = \tan^{-1}\left(\frac{Y_{x21}}{X_{x21}}\right)$$

The relevant components of the frame-2 X and Z unit vectors are illustrated in the diagram on the right. The rates of interest, using $\Delta T \equiv (T_2 - T_1)$, are

$$\dot{X} = \frac{-\Delta X}{\Delta T}$$

$$\dot{Y} = \frac{-\Delta Y}{\Delta T}$$

$$\dot{Z} = \frac{-\Delta Z}{\Delta T}$$

where the minus signs account for optical inversion.

3. Rates for an Entire Scan

The computation described in section 2 was performed on the first two frames for the scan, then frame 2 assumed the role of frame 1, and a new frame 2 was read from the next FITS file in the scan in chronological order. For a scan containing N frames, a table file containing N-1 data rows was written out. Comparison of computed rates to gyro rate values in the header showed excellent agreement in the main scan rate (about Y), and also on the other two axes, but since those rates were small, their limit cycles introduced high-frequency variations affecting most of the significant digits, so those rates could be noticeably different between frame snapshot times and the midpoint times for which the `rates` program computed frame-to-frame values.

4. Sample Output from the Rates Program

The program `rates` read a list of FITS files and performed the computations described above. Output was written to a table file showing the frame-to-frame rates and the averages over the files in the list. A sample output is shown below for the first six pairs of files in a typical survey scan.

```
\AvgRateX   = 0.012525162755226188
\AvgRateY   = 3.7971096839155343
\AvgRatePA  = 0.0000891919185564847
```

Frm1	Frm2	RateX	RateY	RatePA	dT
int	int	real	real	real	real
-	-	arcmin/s	arcmin/s	arcmin/s	s
001	002	0.01014	3.80120	-0.00406	11.00045
002	003	0.01142	3.79531	0.00357	11.00909
003	004	0.01358	3.80119	-0.01163	10.99958
004	005	0.01364	3.79236	0.00523	11.00390
005	006	0.01313	3.79595	0.00916	11.00045
006	007	0.01324	3.79665	-0.00174	10.99958